Antonine Physics A2

Topic 14DTurning Points In Physics

How to Use this Book

How to use these pages:

- This book intended to complement the work you do with a teacher, not to replace the teacher.
- Read the book along with your notes.
- If you get stuck, ask your teacher for help.
- The best way to succeed in Physics is to practise the questions.

There are many other resources available to help you to progress:

- Web-based resources, many of which are free.
- Your friends on your course.
- Your teacher.
- Books in the library.

This Topic looks at some of the most significant historical developments that have made Physics the subject it is today. Physicists in those days were working with primitive technology compared to those of today. Some of the ideas hold true today, while others were plain wrong. Many of the experiments we will look at led to the fundamental concepts which we take for granted in not just Physics, but many other applications. The quantum nature of light and other materials is important for us to see objects, such as the computer screen on which these notes were written.

While some of the ideas we explore may seem outdated or even eccentric, they were the truth to the best of the knowledge and belief. Changing people's ideas is not always easy, especially as some of the protagonists were pretty stubborn. We also explore an experiment that give no results at all but was fundamental to our understanding of the physics of light.

We go on to review what happens when objects travel close to the speed of light.

This option also revises much of the material covered in AS Physics, which you may wish to do before you start studying these notes.

Topic 14D

Option D Turning Points in Physics

1. Fundamental Charges

Tutorial 14D.01 The Discovery of the Electron

AQA Syllabus

Contents

14D.011 Thermionic Emission

If we heat a negatively charged piece of metal, we find that some of the conduction electrons have sufficient kinetic energy to escape from the surface of the wire. It is quite easy to imagine this if we think about a metal wire as a lattice of ions in a sea of free electrons. In effect we are boiling the electrons off. This effect is called **thermionic emission**. This phenomenon had been known about since the middle of the nineteenth century. Experiments on gases at low pressure had revealed a glow around the negative terminal, the **cathode**, and these had been named **cathode rays**. Some physicists had argued that the rays were waves and others had argued that they were negatively charged particles. Indeed, the particles had been named **electrons** by an Irish physicist, George Stoney. Electron comes from the Greek word, εληκτρον, meaning "amber", a resinous material secreted by pine trees.

This was the starting point for Joseph John Thomson to produce his **cathode ray tube** (CRT) in 1897, the descendants of which we used to see every day (*Figure 1*), before TFT TV sets became more common.

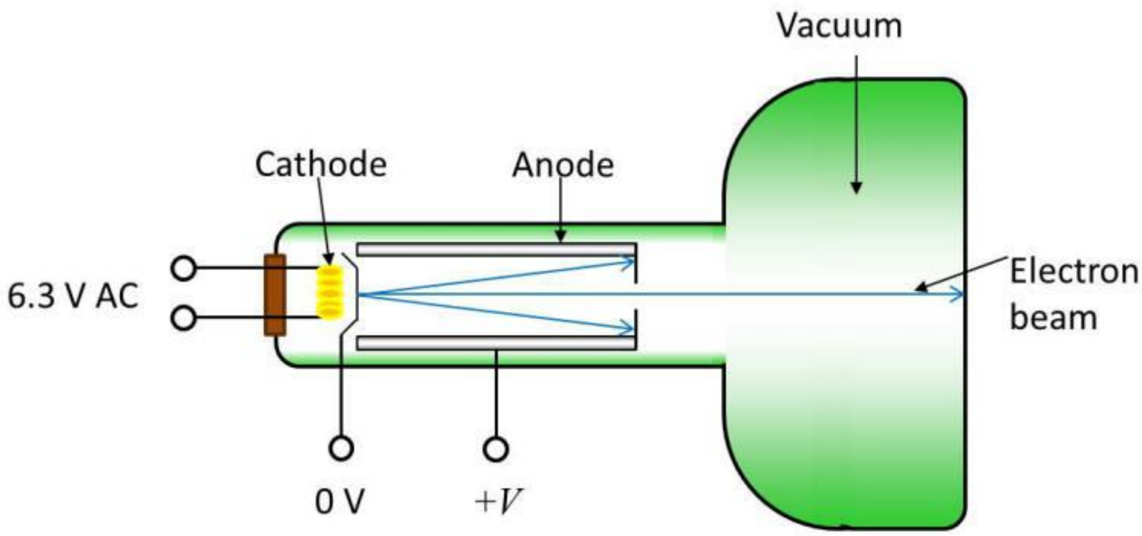


Figure 1 A cathode ray tube (CRT)

Notice that there is a hole in the anode. While most of the electrons boiled off will hit the anode, some will pass through the hole. Hence it is referred to as an **electron gun**. The electrons striking both the anode and the glass at the end of the tube lose most of their energy as heat but can give off some energy as **X-rays**.

The electrons leave the cathode with negligible speed. They are accelerated by the attractive force of the positively charged anode. By the time they leave the gun, the electrons have **energy** eV, where e is the charge on the electron (1.6×10^{-19} C) and V is the anode voltage.

Energy = charge × voltage

$$E = eV \text{ .... } \textit{Equation 1}$$

All the energy in the electron is **kinetic**, so we can say:

$$E_k = \tfrac{1}{2} mv^2 \text{ .... } \textit{Equation 2}$$

So, we can combine the *Equations 1* and *2* to give:

$$eV = \tfrac{1}{2} mv^2 \text{ .... } \textit{Equation 3}$$

Mass of an electron is 9.11×10^{-31} kg

The last answer to Question 14D.01.3 gives an electron speed above the speed of light. This is impossible, because the electron gains mass as its speed approaches that of light. This is a r**elativistic effect** which we will look at later.

14D.012 Charge to Mass Ratio

Thomson used apparatus like this in his classic experiment (*Figure 2*):

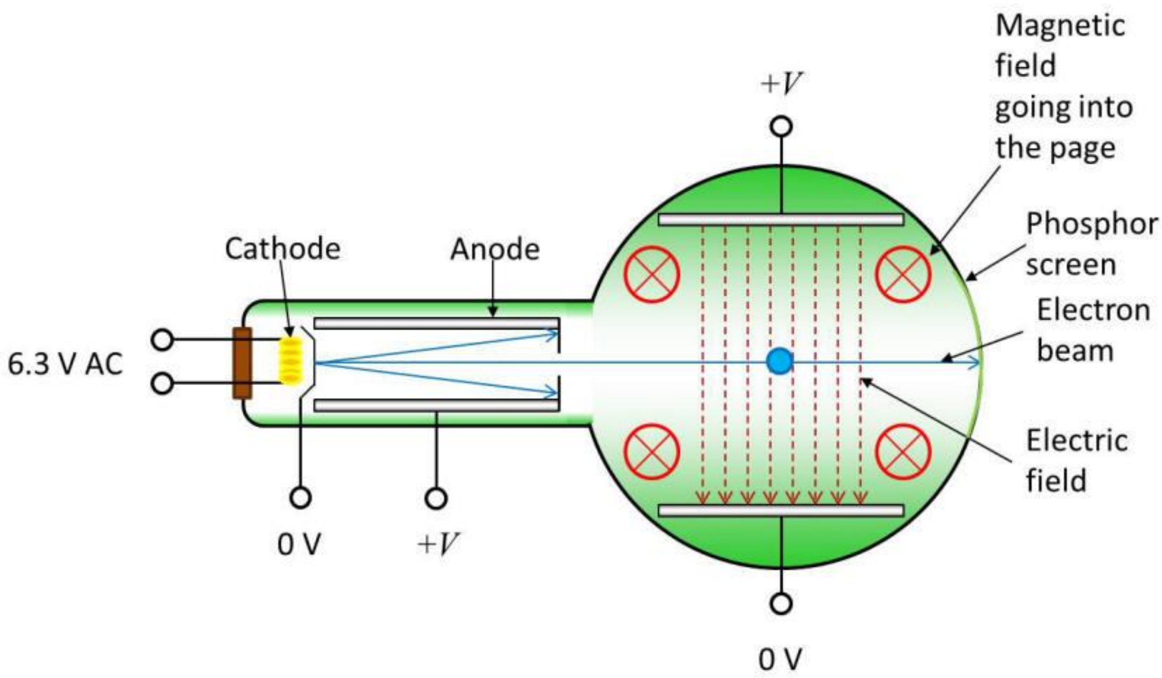


Figure 2 Charge to mass ratio experiment

In this experiment a beam of electrons was passed between a crossed magnetic and electric field. We know:

- In an electric field the electron is attracted to the positive plate. It will accelerate towards the positive plate.
- In a magnetic field the electron is deflected in a circular path at right angles to the magnetic field.

In this experiment we are not interested in the path. Instead, we adjust the values of the magnetic field and electric field so that they are equal in value and in opposite directions.

Consider an electron of charge e that has been accelerated by a voltage V. It passes into a magnetic field of flux density B which combined with an electric field of strength E. The fields are set up so that the resultant force on the electron is zero. This means that the force from the electric field is of equal value but opposite direction to the force from the magnetic field.

We know that:

The Force due to the magnetic field:

$$F = Bev$$

.... *Equation 4*

The electric field is given by:

$$E = \frac{V}{d}$$

.... *Equation 5*

The electric field is force per unit charge:

$$E = \frac{F}{e}$$

.... *Equation 6*

Force due to the electric field is:

$$F = \frac{eV}{d}$$

.... *Equation 7*

Cancelling and rearranging *Equations 4* – 7 give us an expression for the speed:

$$v = \frac{F}{Be}$$

.... *Equation 8*

From *Equation 7* above:

$$F = \frac{eV}{d}$$

.... *Equation 9*

We combine these two (*Equations 8* and *9*) to give:

$$v = \frac{eV}{Bed}$$

.... *Equation 10*

The e terms cancel to give:

$$v = \frac{V}{Bd}$$

.... *Equation 11*

The energy of the electron is given by:

$$E = eV_a$$

.... *Equation 12*

All the energy in an electron is kinetic. The kinetic energy is given by the **accelerating voltage** V_a:

$$eV_a = \frac{mv^2}{2}$$

.... *Equation 13*

We can rearrange *Equation 13* to get the ***e/m*** **ratio**:

$$\frac{e}{m} = \frac{v^2}{2V_a}$$

.... *Equation 14*

We can then get rid of the v term by substituting *Equation 11* into *Equation 14*:

$$v = \frac{V}{Bd}$$

which gives us:

$$\frac{e}{m} = \frac{V^2}{2V_a B^2 d^2}$$ .... *Equation 15*

The terms in the equation are:

- e – the electronic charge, 1.6×10^{-19} C.
- m – the mass of an electron, 9.11×10^{-31} kg.
- V_a – the accelerating voltage (V).
- V – the voltage between the plates (V).
- B – the value of the magnetic field (T).
- d – the separation of the plates (m).

Things are simpler if we make the **accelerating voltage the same as the voltage between the two plates**. If the accelerating voltage and the electric field voltage are the same, we can tidy up our expression to give:

$$\frac{e}{m} = \frac{V}{2B^2 d^2}$$ .... *Equation 16*

There are two terms that need working out:

- the velocity which we can work out from the kinetic energy.
- the B-field which uses the equation $B = 0.716\ \mu_0\ NI/r$. This is not in the syllabus, so we won't pursue it any further.

The term e/m is called the **charge to mass ratio** or **specific charge** of a particle.

The charge to mass ratio can be applied to any particle, although we have seen it applied to the electron and the proton (hydrogen ion). It is constant for a given particle although it will vary between particles.

Always put the sign in for the value of the *e/m* ratio.

14D.013 The Significance of this Discovery

Thomson then went on to show that the e/m ratio was the same whatever gas was used, and he concluded that **all atoms contained electrons**.

He then went on to propose that atoms were made up of electrons embedded in a uniform matrix of protons. The total positive charge was balanced by the total negative charge. If electrons were removed, the remaining ion was left with excess positive charge.

This became known as **Thompson's Plum Pudding model** (*Figure 3*).

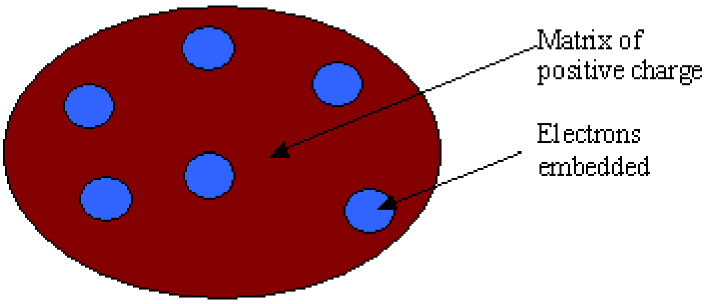


Figure 3 Thomson's plum pudding model of the atom

The Plum Pudding Model was the accepted theory for the structure of the atom until Ernest Rutherford's alpha scattering experiments in 1911. We make think it as rather twee nowadays, but the model worked well to explain things as far as physicists knew at the time.

Positive ions can be accelerated and deflected by a magnetic field. The deflection will depend on the charge to mass ratio. This is the basis of the mass spectrometer.

The Thomson Plum Pudding was disproved by Ernest Rutherford in his famous alpha scattering experiment. The idea of electrons orbiting the nucleus was put forward by Rutherford and Niels Bohr in 1913. This is the model used in A-level physics text.

In those days, nobody knew about the **neutron** until James Chadwick discovered it in 1932, about 20 years after Rutherford discovered the nucleus.

Questions

Tutorial 14D.01

14D.01.1

Explain what these different parts of the Cathode Ray Tube (CRT) do.

(a) Cathode

(b) Anode.

14D.01.2

Why is there a vacuum in the CRT?

14D.01.3

Calculate the kinetic energy and speed of an electron where the anode voltage is

(a) 400 V

(b) 400 kV

14D.01.4

What shape is the path of a moving electron in:

(a) Magnetic Field

(b) Electric Field?

14D.01.5

What is the resultant force in *Figure 2*? How can you tell?

14D.01.6

In *Equation 16* which of these terms is:

(a) Easy to measure directly

(b) Hard to measure directly

(c) Of constant value?

14D.01.7

Show that the specific charge of an electron is -1.76×10^{11} C kg^{-1}. Does the value vary for an electron? Would it be different for a positron?

14D.01.8

What is the specific charge of a proton? How does it compare to an electron?

Mass of a proton = 1.67×10^{-27} kg.

14D.01.9

Is it possible to have anode rays?

14D.01.10

An alpha particle is accelerated at a voltage V. It passes into an electric field formed by the same voltage V across two plates that are 75.0 mm apart. The alpha particle is not deflected when the magnetic field is 0.145 T.

(a) Calculate the charge to mass ratio of the alpha particle.

(b) Calculate the voltage, V, with which the alpha particle was accelerated. Give your answer to an appropriate number of significant figures.

Mass of a proton = 1.67×10^{-27} kg. Electronic charge = 1.60×10^{-19} C.

Tutorial 14D.02 Millikan's Experiment	
AQA Syllabus	
Contents	
14D.021 Millikan's Experiment	14D.022 Stokes' Law
14D.023 Quantisation of Charge	

Now physicists knew that electrons were orbiting the nucleus. But the nature of the electron remained a mystery. They were not the neat little spheres that are found in many a textbook (or, for that matter, these notes). They are quantum beings. The closer you get to the little brutes, the harder they are to catch.

14D.021 Millikan's Experiment

Robert Millikan used a simple but famous experiment that served to confirm the unit electronic charge as 1.6×10^{-19} C. He sprayed oil drops into a space between two charged plates. Each tiny oil droplet was charged up with a negative charge by friction as it left the sprayer. The theory was simple; the attractive electrostatic force between the negatively charged droplet and the positively charged plate would balance out the weight of the droplet. His apparatus was like this (*Figure 4*):

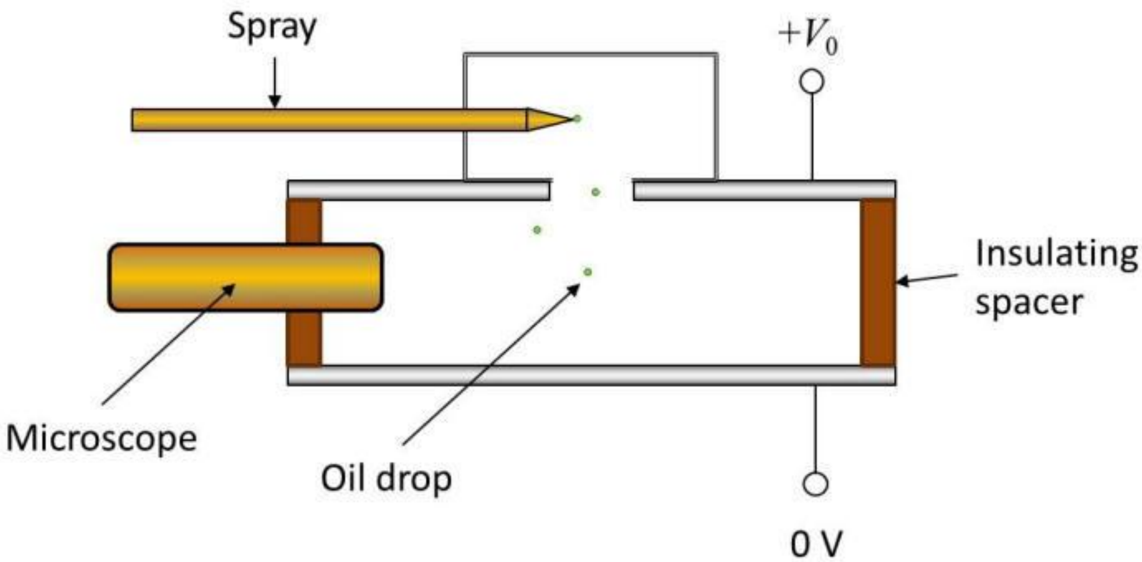


Figure 4 Millikan's oil drop experiment

He would select a particular oil drop and hold it stationary by altering the voltage between the two plates.

The forces on the stationary drop are like this (*Figure 5*):

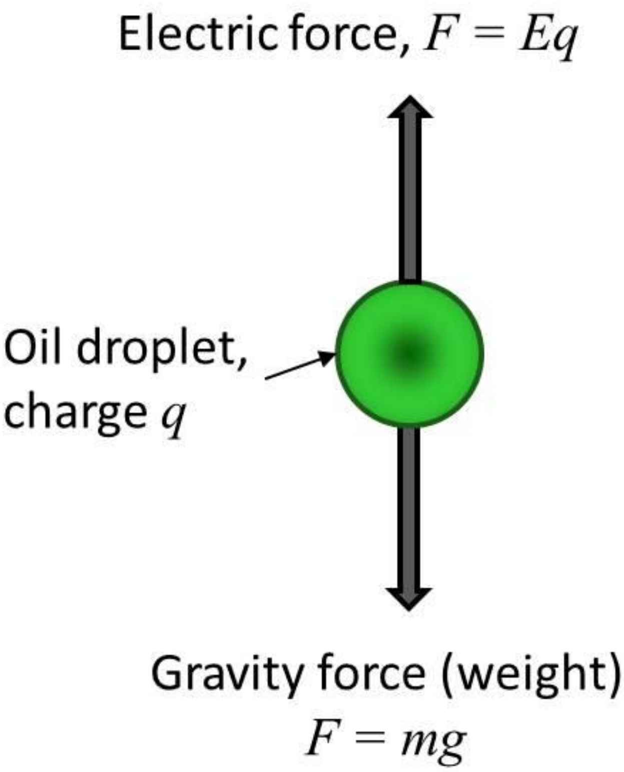


Figure 5 Forces acting on an oil droplet

We know that:

The electric force = electric field × charge

$$F = Eq$$ .... *Equation 17*

The electric field strength in a uniform field,

$$E = \frac{V}{d}$$ .... *Equation 18*

These two (*Equations 17* and *18*) combine to give:

$$F = \frac{Vq}{d}$$

.... *Equation 19*

It doesn't take a genius to see that:

$$mg = \frac{Vq}{d}$$

.... *Equation 20*

14D.022 Stokes' Law

This is not a very satisfactory way because the uncertainty is too great. So, another method was used. Millikan turned off the plates and watched the oil drop. Very quickly the oil drop reached terminal speed. Therefore, we can say that:

mg = drag force

The **drag force** can be worked out indirectly using **Stokes' Law**, which describes the force acting on a sphere falling at terminal speed through a **viscous fluid**. Normally we think of viscous fluids as thick gooey oils; for a tiny oil droplet, air is pretty viscous. The equation that describes Stokes' Law is:

$$F = 6\pi\eta rv$$

.... *Equation 21*

[r - radius of the sphere, v - terminal speed (m s^{-1}).]

The strange looking symbol, η, is "eta", a Greek lower case letter long 'ē', the Physics Code for the **coefficient of the viscosity of a fluid**.

The units for η are N s m^{-2}. For air, $\eta = 1.8 \times 10^{-5}$ N s m^{-2}.

[Note: this model only works for objects falling at low speed. At higher speed, turbulence has an effect.]

In Question 14D.02.2 we looked at the intuitive way of finding the weight by:

Weight = density × volume × g

We assume that the oil droplet is a sphere. We can write this as:

$$mg = \frac{4}{3}\pi r^3 \rho g$$

...........*Equation 22*

So, we can bring in the Stokes' Law equation (*Equation 21*) in by writing:

$$\frac{4}{3}\pi r^3 \rho g = 6\pi\eta r v$$

.... *Equation 23*

Rearranging and cancelling out gives us:

$$r = \sqrt{\frac{9\eta v}{2\rho g}}$$

.... *Equation 24*

By observing the terminal speed, we can work out the radius. From that, we can work out the volume, hence the mass and weight. Although it seems long-winded, this method produces much less uncertainty than attempting a direct measurement of the radius.

Worked example

The data below are from an experiment similar to Millikan's experiment.

- Density of oil = 900 kg m^{-3}
- Pd across the plates = 613 V
- Plate separation = 0.010 m
- Viscosity of air = 1.8×10^{-5} N s m^{-2}

When the voltage between the plates is turned off, the droplet falls steadily a distance of 2.50×10^{-3} m in a time of 22 s. What is the charge? Take g as 9.8 m s^{-2}.

Answer

Work out the speed = distance ÷ time = 2.50×10^{-3} m ÷ 22 s = 1.14×10^{-4} m s^{-1}.

Now work out the radius using the equation:

$$r^2 = \frac{9\eta v}{2\rho g}$$

Substitute in the numbers:

$$r^2 = \frac{9 \times 1.80 \times 10^{-5}\ \mathrm{N\,s\,m^{-2}} \times 1.14 \times 10^{-4}\ \mathrm{m\,s^{-1}}}{2 \times 900\ \mathrm{kg\,m^{-3}} \times 9.8\ \mathrm{m\,s^{-2}}}$$

Therefore:

$$r^2 = 1.05 \times 10^{-12}\ \mathrm{m^2}$$

Remember to take the square root.

$$r = 1.02 \times 10^{-6}\ \mathrm{m}$$

Mass = volume × density:

$$m = \frac{4}{3}\pi r^3 \times \rho$$

Substitute:

$$m = \frac{4}{3}\pi \times (1.02 \times 10^{-6}\ \mathrm{m})^3 \times 900\ \mathrm{kg\,m^{-3}} = 4.00 \times 10^{-15}\ \mathrm{kg}$$

We know that weight = force from the electric field

$$mg = \frac{qV}{d}$$

Rearranging gives us:

$$q = \frac{mgd}{V}$$

Substituting:

$$q = \frac{4.00 \times 10^{-15}\ \text{kg} \times 9.8\ \text{m s}^{-2} \times 0.010\ \text{m}}{613\ \text{V}}$$

Therefore:

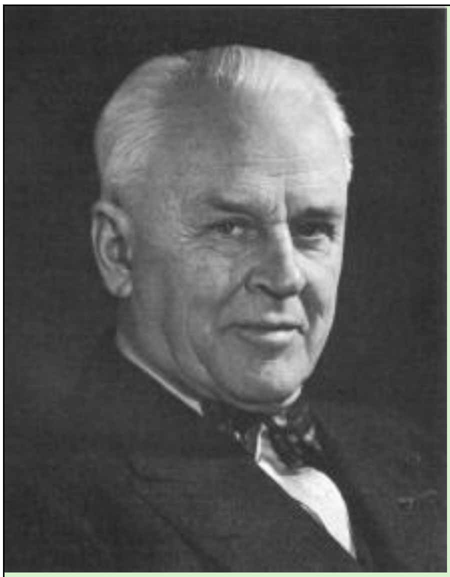
q = **6.4×10^{-19} C**

14D.023 Quantisation of Charge

The answer above is a charge of magnitude $4e$ (where e = -1.602 × 10^{-19} C).

The important finding from this experiment was that the charge was always **a whole number multiple of 1.6 × 10^{-19} C**. Therefore, the electron charge came in definite amounts (**quanta**); therefore, it is said to be **quantised**. Electrons are indivisible, i.e. fundamental particles of matter.

Once we know the charge on an electron, a simple calculation will tell us the mass of the electron, 9.11 × 10^{-31} kg.



Robert Andrews Millikan (1868 - 1953) was an American physicist who won the Nobel Prize for this work and other work he did on the photoelectric effect. However, his first degree was not in Physics, but in Classics. He was a promising scholar in Latin and Greek. During his final year his Greek professor asked him to take the foundation Physics class, to which he, not unnaturally, objected on the grounds that he knew no physics. The professor told him, "If you can do well in Greek with me, you can teach physics." After warning the professor that he would not be responsible for the outcome, Millikan bought a text book, and kept three pages ahead of his class, discovering that he really enjoyed it. And that rubbed off onto his class.

Gaining a doctorate in 1895, Millikan soon became professor at Chicago University. In 1908 he started work on his oil drop experiment.

Millikan enjoyed the teaching aspect of his work. He wrote a number of text books that were well ahead of their time, that encouraged students to "think in physics", rather than to apply formulae in a mechanical manner.

This experiment won Millikan the Nobel Prize in 1923.

Questions

Tutorial 14D.02

14D.02.1

What are the forces acting on the droplets between the plates? What is the resultant force?

14D.02.2

How might you find the weight of the drop?

14D.02.3

What forces are acting on an object at terminal speed? What is the resultant force?

14D.02.4

A small metal sphere of radius 0.50 mm has mass 1.0×10^{-3} kg is dropped into oil of which the viscosity is 0.36 N s m^{-2}. What is the terminal velocity at which it falls?

14D.02.5

In an experiment to determine the charge on a charged oil droplet, the droplet was held stationary in a vertical electric field of strength 57 kV m^{-1}. After the field was switched off, the droplet fell at a steady speed, taking 18.3 s to fall through a vertical distance of 2.0 mm

Viscosity of air = 1.80×10^{-5} N s m^{-2}

Density of oil = 970 kg m^{-3}

g = 9.8 m s^{-2}

(a) Calculate the speed of the droplet as it falls.

(b) Show that the droplet's radius is 9.7×10^{-7} m

(c) Calculate the charge of the droplet.

(d) Compare this to the electronic charge. What does it suggest?

2. Light - Particle or Wave?

Tutorial 14D.03 Is light a Wave?

AQA Syllabus

Contents

Isaac Newton (1642 – 1727) did many of the early experiments on light in the Seventeenth Century. His argument was that light was a stream of particles. This was nothing new; the Ancient Greek philosopher Democritus had proposed that objects were visible because of the swarm of particles that they put into the air.

14D.031 Light as a Particle

Newton's evidence was that:

- Objects cast sharp shadows; if they were a wave, the shadows would be fuzzy.
- Light passed through a vacuum; there was no material for a wave to propagate through.

His assumption was that particles travelled at a constant velocity except when near the boundary between two substance when unbalanced forces act on them.

In **reflection** the velocity perpendicular to the surface was reduced to zero, then increased to the original value in the opposite direction. The velocity component parallel to the surface was unchanged (*Figure 6*).

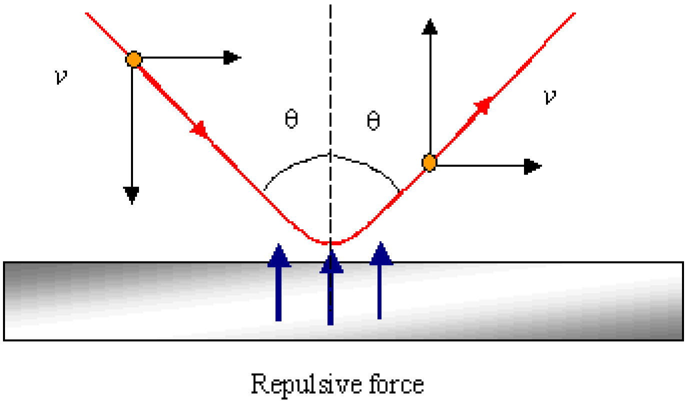


Figure 6 Reflection of a particle of light

In **refraction** the velocity perpendicular to the surfaces was increased by an attraction force. The velocity component parallel to the surface was unchanged. The light ray was bent towards the normal (*Figure 7*).

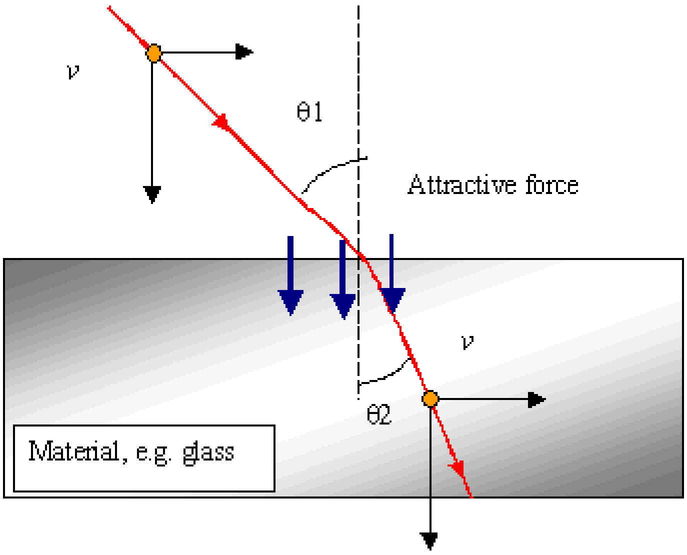


Figure 7 Refraction of a particle of light

The splitting of white light into the colours of the rainbow was explained by the difference in forces on the particles of different colours.

There are other limitations:

- The theory could not explain partial reflection or refraction.
- It could not explain interference.
- It seemed that there was both attraction and repulsion at some boundaries.

Another problem is that if there is an attractive force on the perpendicular movement, the path is not the straight line that we observe, it's a parabola, just like in projectile motion.

14D.032 Light as a Wave

At the same the same time as Newton, a Dutch physicist **Hans Christiaan Huygens** (1629 – 1695) proposed that light was a wave. His view was prompted by the observation that beam of light cross each other without scattering. If they were particles, there would be collisions between the beams.

His assumption was that light waves spread in all directions at a constant speed in a material called **ether**. Ether permeated everything including a vacuum.

14D.033 Huygens' Construction

Huygens (pronounced "*Harkens*") proposed models that consisted of **plane wavefronts** which consisted of **secondary wavelets.** We tend to think of light passing as **rays**. A ray describes the direction of travel of a light wave. The way that Huygens describes light was as a set of plane wavefronts. The wave fronts were at 90 ° to the ray (*Figure 8*):

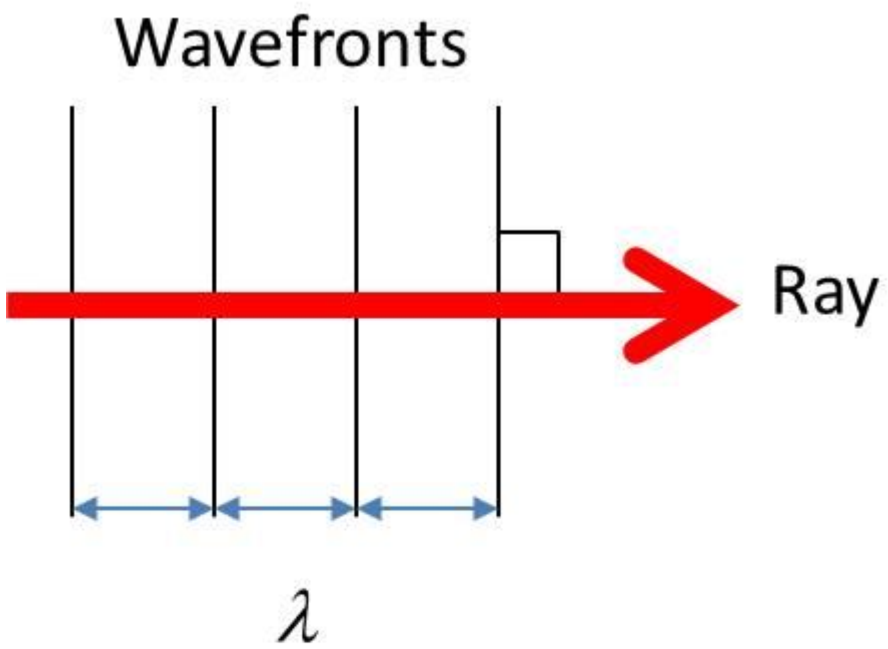


Figure 8 Huygens' construction of plane wavefronts

Huygens had observed that a disturbance at single point source resulted in **circular propagation** of the waves (*Figure 9*):

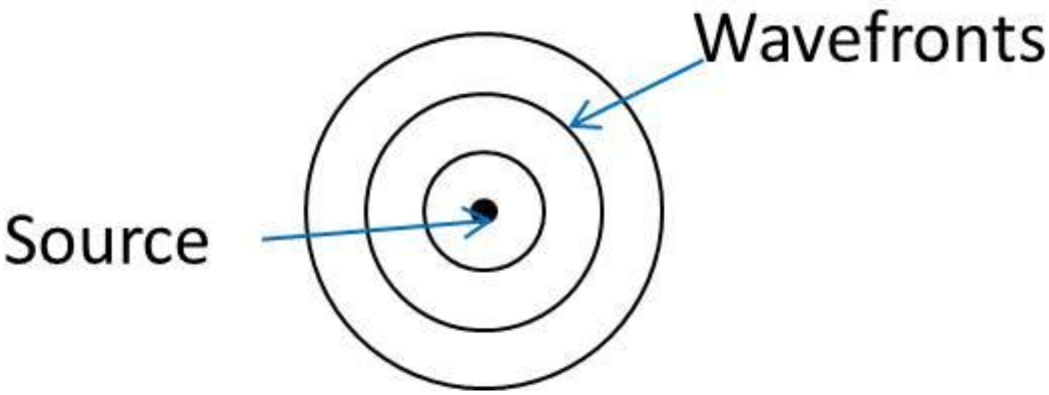


Figure 9 Circular wavefronts from a point source

In his principle he said that plane wavefronts were produced by many sources producing secondary wavelets. A plane wavefront was produced from a line of **point sources** each producing secondary **wavelets**:

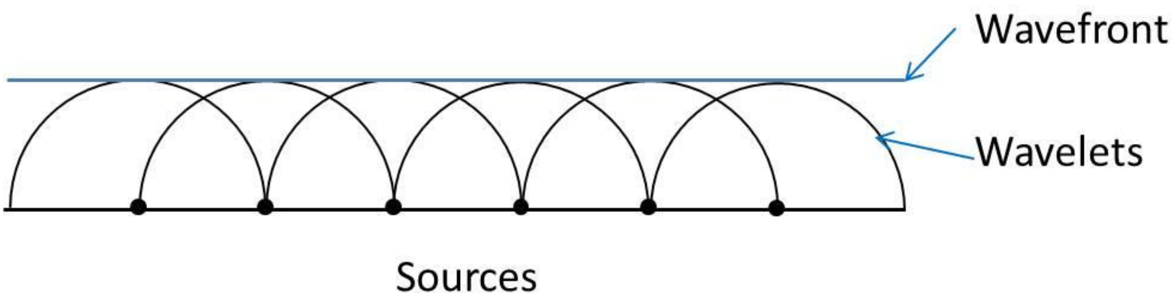


Figure 10 Plane wavefronts produced from many point sources

Only a few sources are shown here. The wavefront would not be perfectly straight. If there were lots of sources, the wavefront would be straight. On the wavefront that is propagating, secondary sources produce more wavelets to make another wavefront. See *Figure 11*.

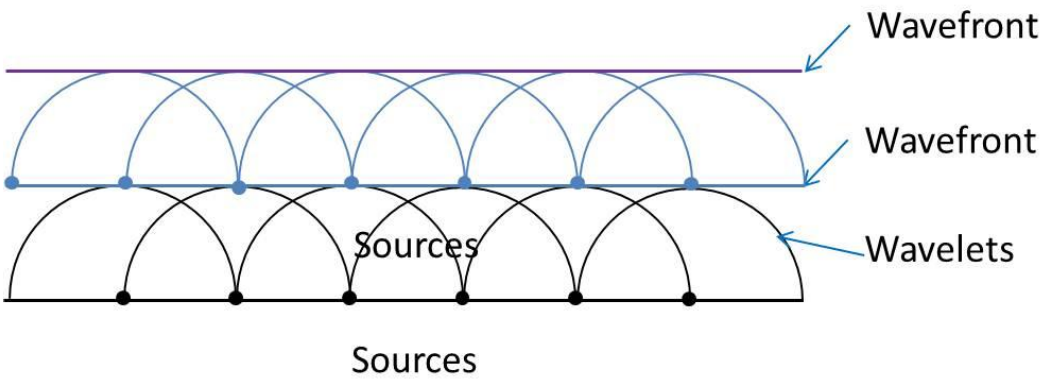


Figure 11 A second wavefront is produced from secondary wavelets from the wavefront behind

The same principle can be applied to curved wavefronts (*Figure 12*):

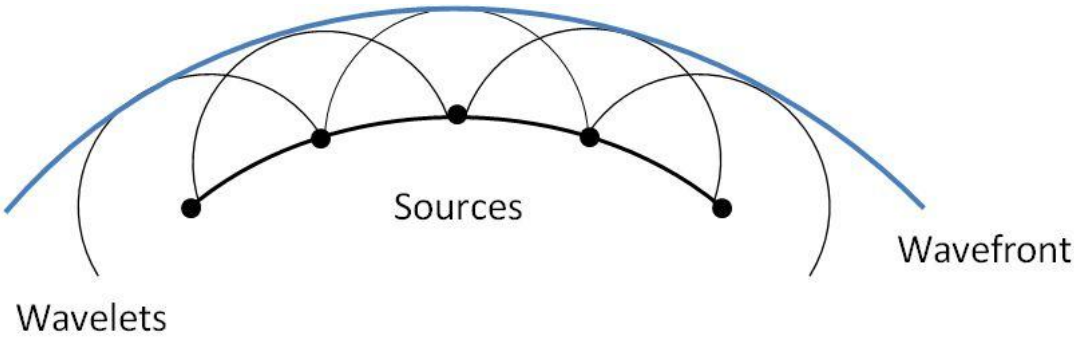


Figure 12 Curved wavefronts are made from point sources that are on a curved wavefront

Huygens' Principle can be applied to **reflection**, **refraction** and **diffraction**.

The patterns seen are rather like those observed in a **ripple tank**, which you will have seen at times in your physics lessons.

14D.034 Reflection

The mechanism for **reflection** is like this (*Figure 13*):

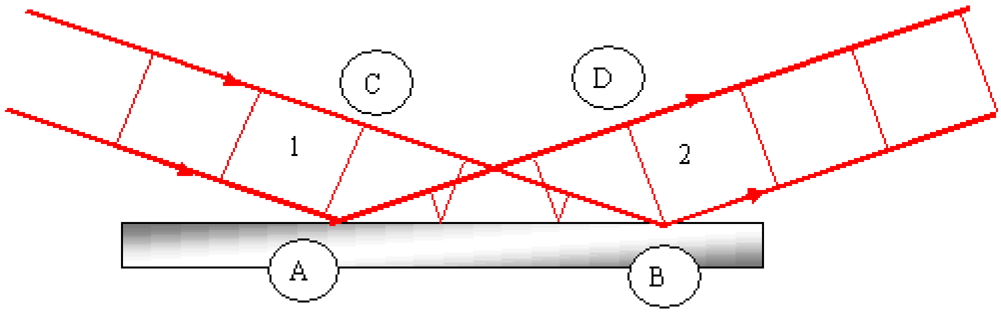


Figure 13 Huygens' Construction for reflection

- Wavefront 1 reaches point A.
- A wavelet from A starts to spread out.
- When the incident wavefront reaches B, the secondary wavelet from A has reached D, giving a new wavefront 2.
- Geometry can be used to show that the wavefronts make equal angles to the boundary, so that **angle of incidence = angle of reflection**.

The behaviour of the wavelets is shown in the picture below (*Figure 14*):

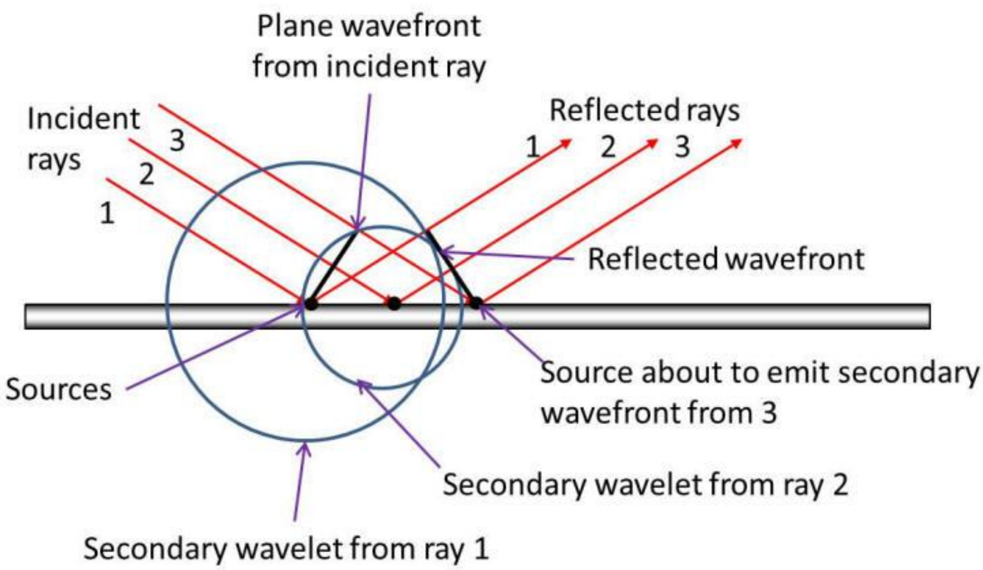


Figure 14 Wavelet formation in reflection

14D.035 Refraction

Huygens explained **refraction** in a similar way (*Figure 15*).

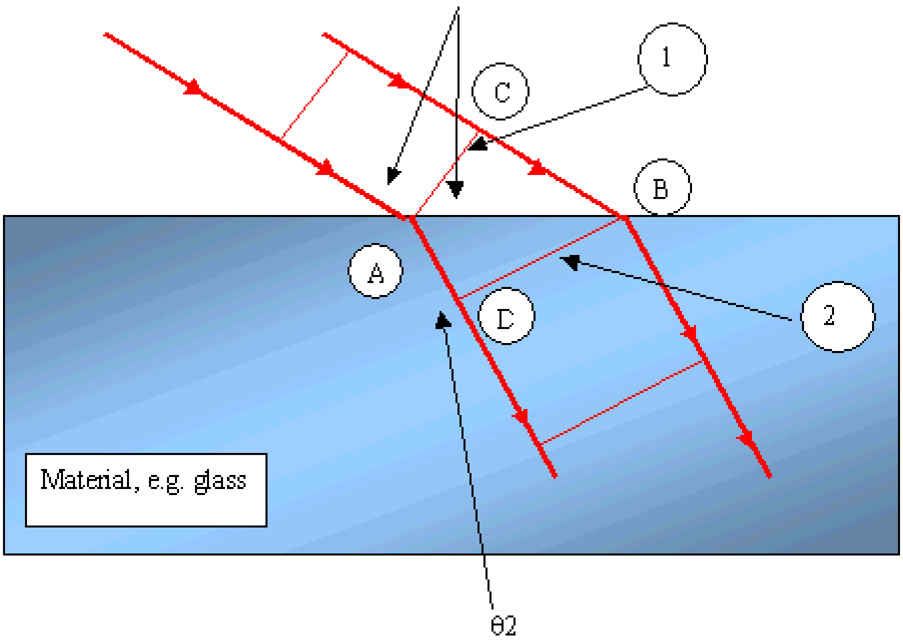


Figure 15 Huygens' Construction for refraction

- Wavefront 1 reaches A.
- Wavefront from A starts to spread out.
- When incident wavefront reaches B, secondary wavelet from A has travelled a shorter distance to reach D.
- It gives a new wavefront 2.
- As a result, the wave path bends towards the normal.

The diagram below (*Figure 16*) shows the idea with the secondary wavelets:

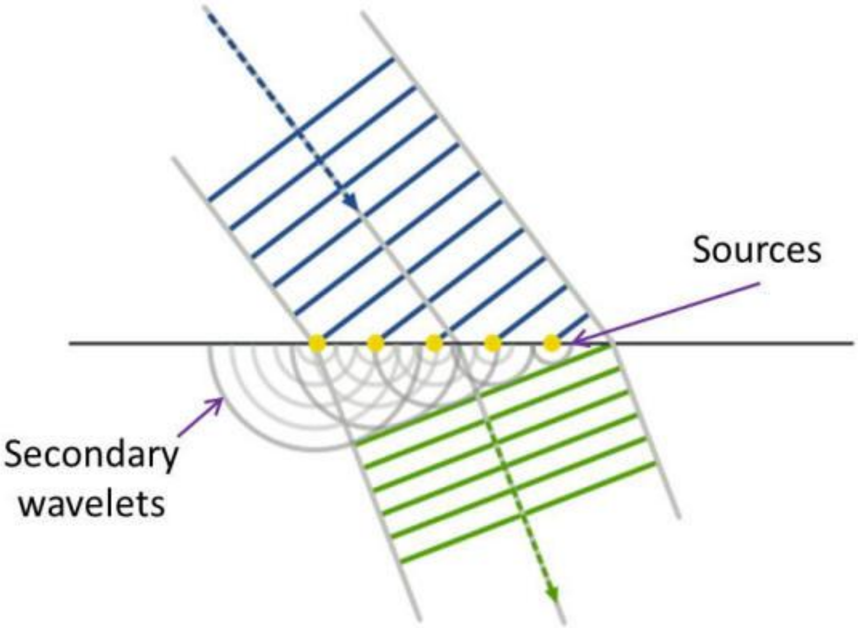


Figure 16 Secondary wavelets in refraction (Image from Arne Nordman, Wikimedia Commons (adapted))

His theory showed that the wave speed in the material was less than the wave speed in air.

These observations are very similar to what we see when we study water waves.

Huygen's theory did not discuss ideas of frequency and wavelength. Light was though to emitted in pulses of energy. There was no mathematical theory of continuous waves. Nor had diffraction of light been noticed.

Although the wave behaviour of light was supported by many eminent scientists, it was rejected out of hand by Newton, who was the **pre-eminent scientist with a considerable reputation**. Newton was not a pleasant man, cantankerous and desperately self-opinionated (Reminds you of anybody?). Anyone who stood up to him risked having his work and character ridiculed. That happened to Huygens, whose work remained overlooked for many years. Also, **waves could not pass through a vacuum**, so it was thought. However, on the continent, Huygen's theory carried more weight.

There was little evidence either way and the debate ran on for many decades until the Nineteenth Century, until Thomas Young (1773 – 1829) performed his **Double Slit** experiment in 1801.

14D.056 Diffraction

Huygens' construction can also be used to explain **diffraction** (*Figure 17*):

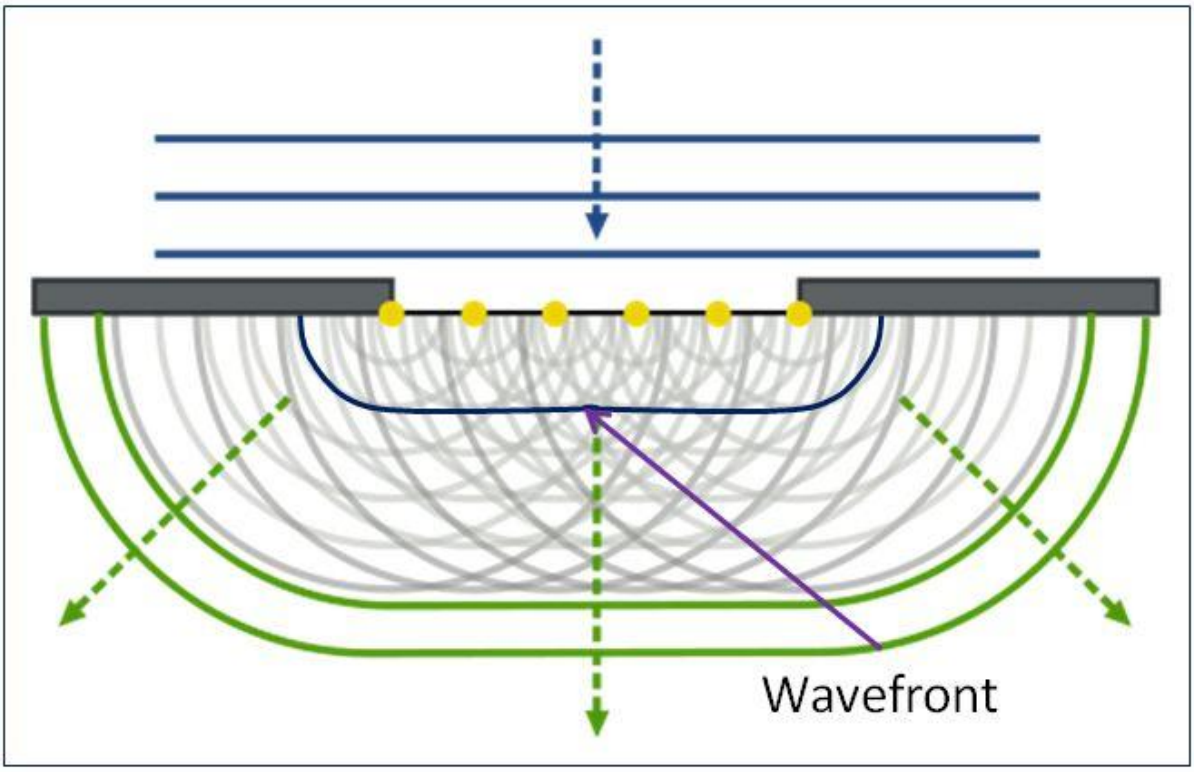


Figure 17 Huygens' Construction in diffraction

The secondary wavelets come from the sources that are in the gap. Wavefronts form along the corresponding secondary wavelets. At the ends, the wavefront is curved, following the pattern of the circular secondary wavelet.

Huygens' construction explains how light can be propagated as a wave. It still provides a good foundation for classical optics, although it is a static model, rather than dynamic model. It provides a snapshot, rather than video, of the propagation. In those days, the wave equation had not been worked out.

14D.037 Young's Double Slits

In this experiment, Thomas Young demonstrated **interference**. Part of the problem of demonstrating interference using two light sources is that getting **coherent** waves is impossible. However, if the ray from one source is split into two, by its nature, the ray consists of coherent waves. Also getting pure **monochromatic** light was not easy, even with coloured filters.

Nowadays it's very easy with a **laser**, which always gives out coherent monochromatic light.

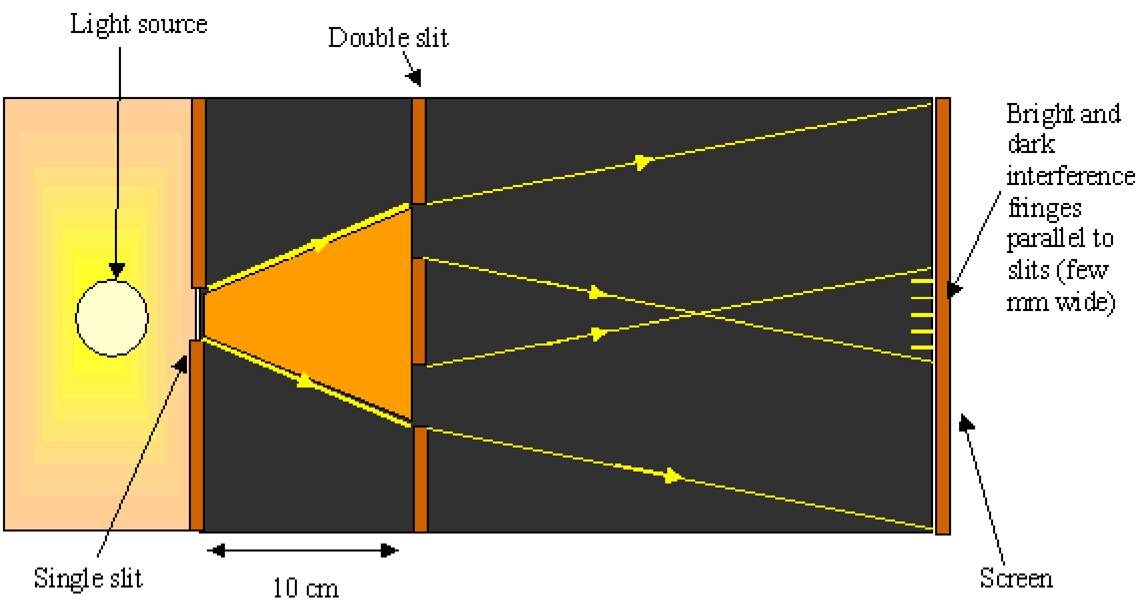


Figure 18 Young's double slit experiment

In his day Young could only use a dim light source such as a candle or oil lamp (*Figure 18*). However the effect could be seen convincingly, and the experiment became a turning point in the debate between those who considered light as a wave. **There was no way that it could be explained by particles**. On the other hand, it could be **explained easily by wave theory**.

- Light waves reach the screen after travelling different paths.
- At various places across the screen the waves are in phase or out of phase.
- Waves that are in phase add up to give a bright fringe; waves out of phase cancel out to give a dark fringe.

Although this was clearly a wave phenomenon, there was some delay before it was whole-heartedly accepted, which happened when a good mathematical argument was worked out. Even then not all were convinced.

In 1850 the speed of light was measured in air and in water. The speed of light in water was found to be lower than that in air, which:

- gave the answer predicted by the wave theory of refraction…
- …and contradicted the particle theory.

Then the particle theory was finally abandoned in mainstream Physics thinking.

14D.038 Electromagnetic Waves

In 1865 the theoretical physicist James Clerk-Maxwell (1831 – 1879) predicted that an oscillating electric field would cause a magnetic field to oscillate and vice versa. By dint of rigorous mathematical analysis, he predicted that the waves would propagate as a transverse wave and gave a formula for their speed.

$$c = \frac{1}{\sqrt{(\mu_0 \varepsilon_0)}}$$

………….. *Equation 25*

- μ_0 (pronounced "mu-nought") is the Physics code for **permeability of free space**. It is a constant and has a value of $4\pi \times 10^{-7}$ H m^{-1}. It an important component in the mathematical analysis of magnetic fields and induction.
- ε_0 - (pronounced "epsilon-nought") is the Physics code for the **permittivity of free space**. It is a constant and has the value 8.85×10^{-12} C^2 N^{-1} m^{-2}. It is an important part of electric field phenomena.

It had been discovered in 1831 that a changing magnetic field always induced a voltage. It would be reasonable to suppose that there would be an electric field associated with this voltage. Maxwell also considered that it would be reasonable to say that a changing electric field would induce a magnetic field, which in turn would produce a changing electric field and so on. From this he concluded that the electromagnetic wave where the electric field and the magnetic fields are at right angles to each other (*Figure 19*).

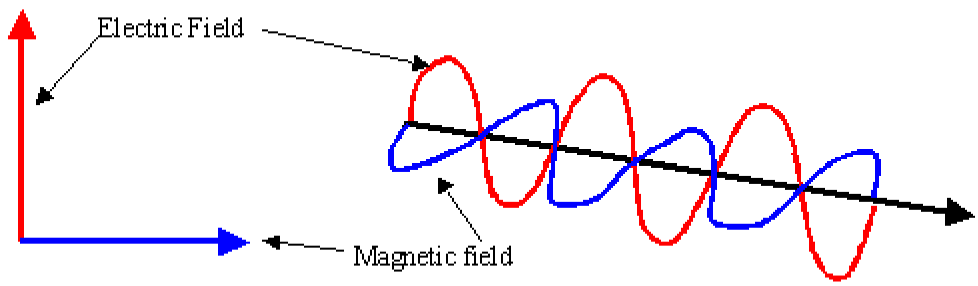


Figure 19 Electromagnetic wave

The diagram above shows:

- a transverse wave
- with an electric field vector
- and a magnetic field vector at right angles.

Electromagnetic waves can be **polarised**. When the electric field vector is vertical, the wave is vertically polarised; when it is horizontal, the wave is horizontally polarised.

14D.039 Radio Waves

The electromagnetic wave remained a theoretical concept until their existence was demonstrated by a German Physicist Heinrich Hertz (1857 – 1894). He set up this lethal looking apparatus (there was no Health and Safety at Work Act at the time):

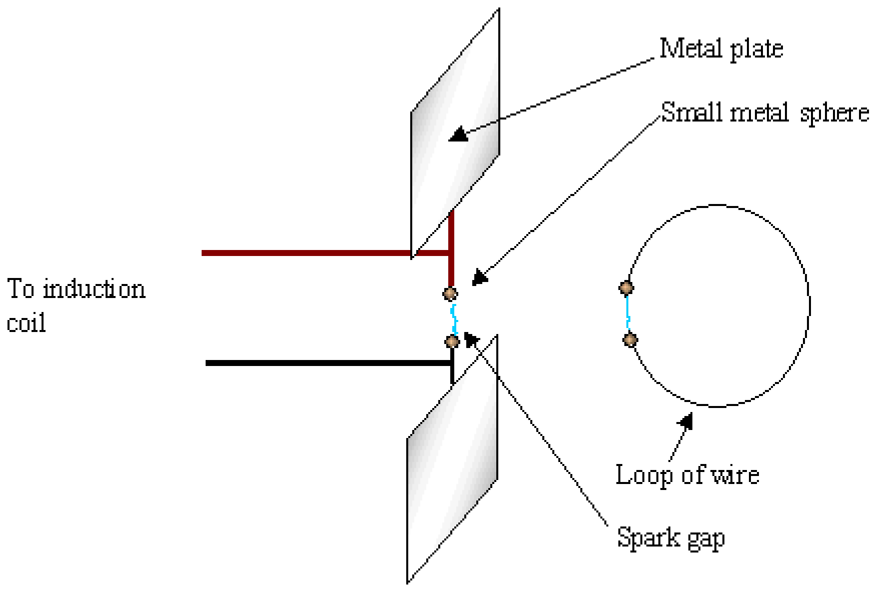


Figure 20 Apparatus used by Heinrich Hertz

- The induction coil produces a very high voltage.
- The electric field strength between the spheres is strong enough for the insulating properties of air to break down (approximately 3000 V mm^{-1}).
- The effect was intensified if the radiation was reflected using parabolic reflectors on both the transmitter and receiver,
- When the spark jumps, it generates a spark that produces a high frequency **damped electrical oscillation**.

The high frequency oscillations induce a voltage in the loop which was sufficient to cause a small spark to jump. The spark could be made more powerful if the loop was put at the focal point of a concave mirror. If he put the loop on its side, the effect was not seen at all, indicating that the waves were **polarised**. The two pictures (*Figures 21* and *22*) below explain in simple terms why the polarisation happens.

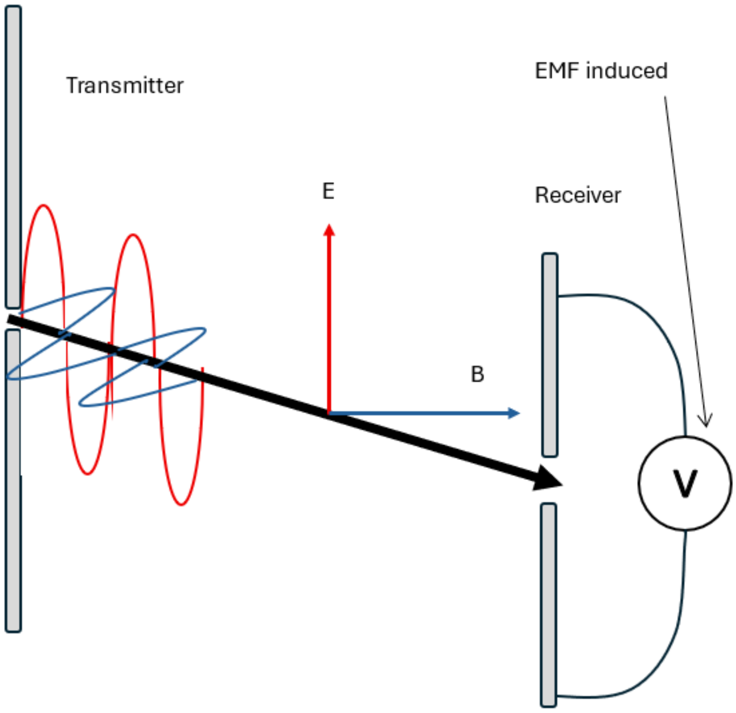


Figure 21 A vertical wire has an EMF induced in it by a vertically polarised EM wave

The E field exerts a force $F = Eq$ on the electrons, making them move up and down. An emf is induced. Also, the magnetic component of the waves is at 90 degrees to the aerial. An emf is induced by Faraday's and Lenz's Laws.

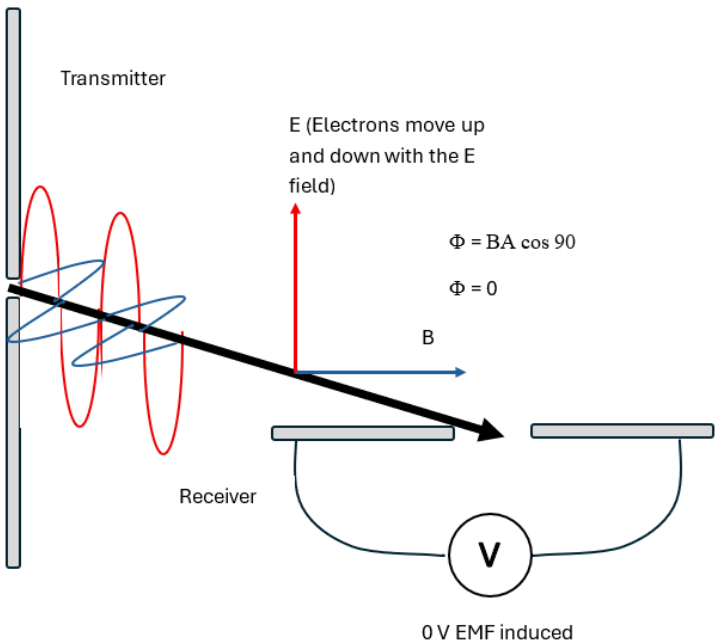


Figure 22 The horizontal wire has a zero EMF induced

If the aerial is moved through 90 degrees, the movement of electrons remains up and down, but this movement is 90 degrees across the wire. Therefore, no emf is induced. Additionally, the B-field component is parallel with the wire.

$$\Phi = BA \cos 90 = 0.$$

The alternating wave is not sinusoidal. The EMF induced by the induction coil rises and falls exponentially, as current in an inductor rises and falls exponentially. The reasons for this are not on the syllabus. If we measure the output from the receiver with a CRO, it would be very messy as there is sparking.

Another experiment that Hertz carried out was study the standing wave nature of radio waves. He moved his detector between his transmitter and a flat metal sheet. He noticed that there were points at which the sparking was non-existent, or very weak. The weak points coincided with the nodes of a standing wave pattern. You have probably seen a similar experiment carried out using 3 cm waves.

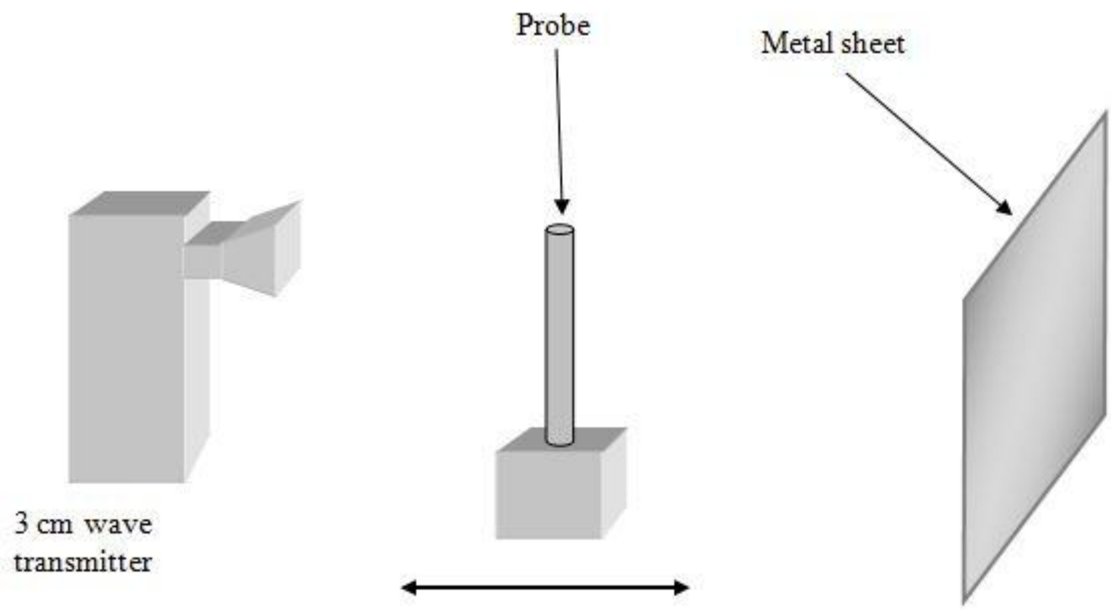


Figure 23 Standing waves in 3 cm wave apparatus

Hertz went on to work out the frequency on the assumption that the effect was due to electrical resonance. He also set up standing waves and found the wavelength between nodes. This allowed him to determine the speed of the waves, getting a very similar result to what Maxwell had predicted in his theoretical calculations. However, Hertz could not think of any earthly use for this result, other than that it was an interesting physics curiosity.

This experiment formed the basis for experiments by Guillermo Marconi, the son of an Italian Count, into **wireless telegraphy**.

It had been clinched. **Light was a wave**.

It is easy to dismiss the early ideas as ludicrous. Modern physicists have the truths based on years of painstaking research by the world's most eminent physicists, which are handed down from generation to generation. Each generation of teachers has its set of books on which their material is based; the style will change, but the essential truths will not. The early physicists had none of these; their work was based entirely on observation from pretty primitive technology. It is easy to pick up misconceptions where there is no bedrock of underlying truth. The theories produced were presented in total good faith. The early physicists were men of integrity, even if they were at times head strong. Obnoxious though he was, Newton produced the laws on which space flights are based today.

Contrast that to some scientists today who, for whatever reason, have sacrificed integrity in order to fulfil an unrealistic target set by a manager, or worse still to make a quick buck.

Isaac Newton (1642 - 1727) was a brilliant mathematician, physicist, and unorthodox theologian. But he was a most unpleasant man. He was rude, bad-tempered, self-centred, self-opinionated, and held on to grudges. As a baby he was very small and would "fit into a quart jug" - just over a litre. His father died when he was three and his mother re-married. He loathed his stepfather. In his late teens, Newton's mother withdrew him from the King's School in Grantham, and tried to make a farmer of him, which he hated. On the persuasion of the Headmaster, he was allowed to return and became a top-flight student (to annoy a school bully). He was admitted to Cambridge University in 1661.

At Cambridge, he was initially an undistinguished student, but later his talents were recognised. Although he had to return home in 1665 as Cambridge University closed because of bubonic plague, he was obsessive in his studies, producing a very wide range of work. At first, he was reluctant to publish these as he was afraid, so it seemed, of the criticism that might have come his way. This shyness was not to feature in his middle age. He fell out with many other academics, often publicly and with much bitter argument. Newton not only tolerated no rivals but also saw to it that he ruined their reputations. One such was Robert Hooke (of Hooke's Law), who himself was, by all accounts, almost as obnoxious as Newton. Newton is also said to have rubbished the work of Christian Huygens, whose work remained unrecognised for many years. Newton was quick to take the credit for himself when something that went well. When something went badly, he rapidly off-loaded the blame onto others.

In 1689 Newton was a member of parliament, although it seems he achieved little there. In 1717 he was appointed Warden of the Royal Mint, and was conscientious in his duties, leading a number of people to be sentenced to death of making counterfeit coins. He also had himself appointed as a magistrate.

There are a number of myths, the most famous being that he built with his own hands the Mathematical Bridge in Cambridge, which was said to have been constructed with no nails. (This would be consistent with Newton's being a fine craftsman.) It was actually designed twenty years after his death. It is also said that when his cat had kittens, he cut six extra holes in the door to enable each kitten to have its own entrance. That would not have prevented Newton from taking credit for it. Other stories tell that he was completely hopeless at practical tasks. Some people suggest that he had the high functioning autistic trait of Asperger's Syndrome, although some of his more eccentric behaviour of later life may have been due to mercury poisoning, as a result of his many experiments with alchemy.

Questions

Tutorial 14D.03

14D.03.1

What happens to the resultant velocity as a result of the change that Newton proposed? See *Figure 7*.

Is it consistent with what you know about the speed of light in a material?

14D.03.2

When a wave is reflected, its phase changes by 180°. How do you think this diagram is consistent with this observation?

14D.03.3

Is this theory of refraction (*Figures 15* and *16*) consistent with what you know already about refraction?

14D.03.4

Why was the wave nature of light slow to catch on?

14D.03.5

What is meant by coherent waves?

14D.03.6

What made physicists believe that light was a wave rather than a particle?

14D.03.7

Show that:

$$c = \frac{1}{\sqrt{(\mu_0 \varepsilon_0)}}$$

is about 3×10^8 m s^{-1}.

14D.03.8

What would the charge time graph for the electrical oscillation in Hertz's apparatus look like? (Hint: It's like a spring)

Tutorial 14D.04 Light is a Particle

AQA Syllabus

Contents

Light was a wave. That was that. Well… er… no.

The **photo-electric effect** had initially been described by Hertz in 1887, and developed further by another German, Hallwachs. A negatively charged zinc plate would discharge when exposed to ultra-violet light; a positively charged plate would not. Also, the plate would not discharge in bright red light, for example from a laser, but it would in dim UV light. You may wish to revise the photo-electric effect in Topics 2 and 3.

14D.041 The Ultra Violet Catastrophe

Study of **black body radiation** provided results that could not be predicted by classical physics. You can follow the idea of black body radiation up in **Topic 14A Astrophysics**. A black body is defined as a perfect emitter of radiation. At thermal equilibrium the black body will emit radiation at all wavelengths. As frequency of the radiation increased, the more energy would be given out. If all energies were calculated at all frequencies, the energy becomes infinite. This would breach the Law of Conservation of Energy. This problem was called the **ultraviolet catastrophe**.

The model was that a black body radiator would set up electromagnetic stationary waves in a cavity, like this (*Figure 24*):

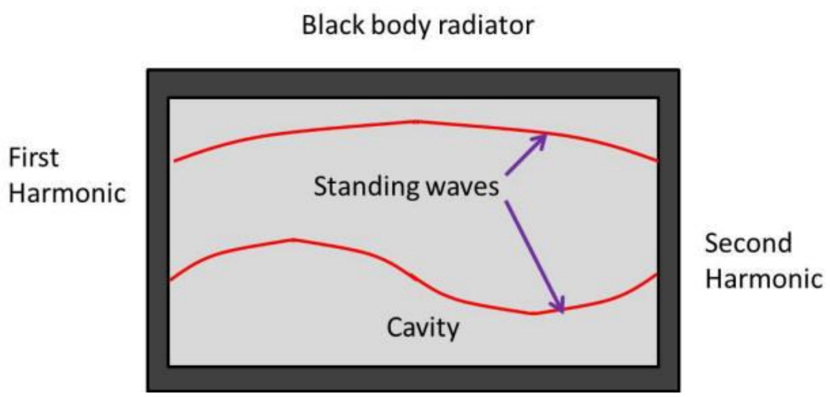


Figure 24 Electromagnetic stationary waves in a black body radiator

It's like the sound box of a musical instrument in which standing sound waves can be set up at the first harmonic, f_0, or the second harmonic $2f_0$, or any whole number multiple of the first harmonic. There is an equal probability of all the different harmonics, this gives the musical instrument its characteristic sound quality. The same applies to a black body radiator, except that the standing wave were electromagnetic, and the wavelengths were much shorter. In the diagram above, there are two modes. In *Figure 24*, we see that the standing waves travel across the length of the cavity. However, they can set up standing waves between any two points like this (*Figure 25*):

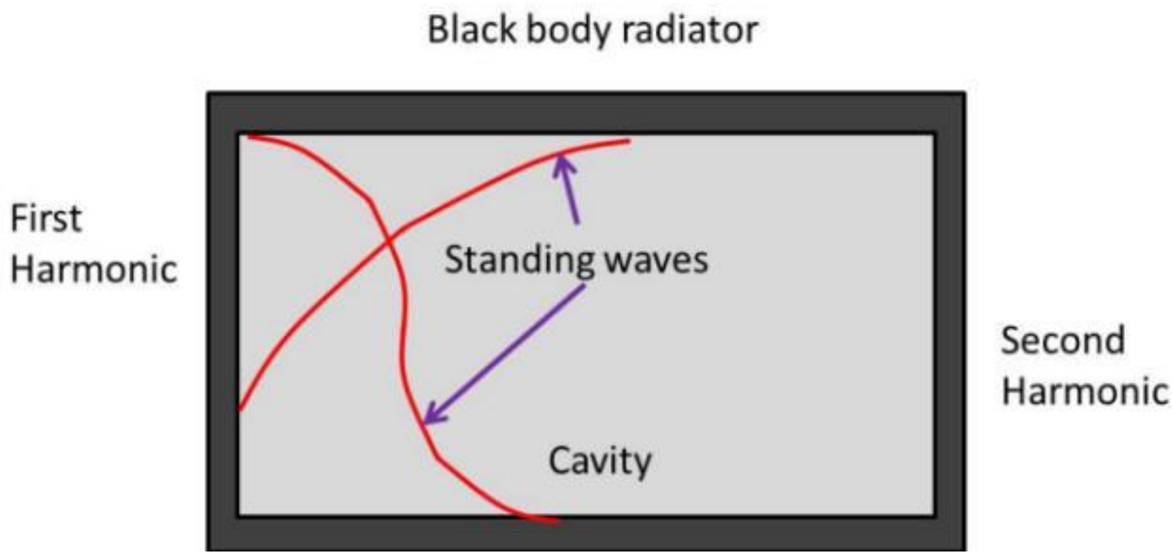


Figure 25 Standing waves can occur between any two points in a black body radiator

With higher frequencies, more standing waves are possible. In fact, the number of standing waves (or modes) predicted varies as the square of the frequency, according to classical wave theory. The Rayleigh-Jeans Law was worked out in 1905 by John William Strutt, Lord Rayleigh, (1842 - 1919) and Sir James Hopwood Jeans (1877 - 1946). The relationship they came up with was:

$$B_\lambda T = \frac{2ckT}{\lambda^4}$$

.... *Equation 26*

[B_λ - spectral radiance; T - temperature (K); c - speed of light (m s^{-1}); k - Boltzmann's Constant (m^2 kg s^{-2} K^{-1}); λ - wavelength (m)]

You are NOT expected to use this equation - it is here for illustrative purposes.

The formula, called the **Rayleigh-Jeans Equation** worked for long wave radiation, but not short. The graph was like this (*Figure 26*):

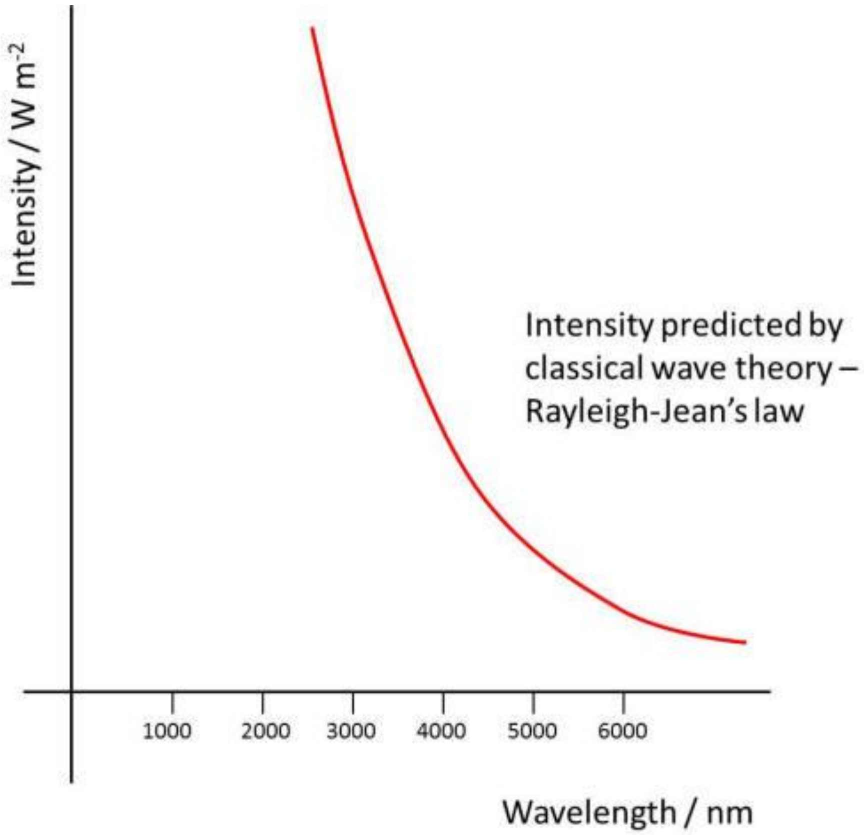


Figure 26 Graph of the Rayleigh - Jeans equation

This graph suggests that the intensity at very small wavelengths tends to infinity. Energy is being created, which cannot happen. The model had broken down.

The German physicist Max Planck (1858 – 1947) tried to explain the observations in terms of classical physics but could not produce a convincing explanation. Radically rethinking the problem, he concluded that **classical physics does not always apply at the atomic level**. Instead, he then proposed that **energy was radiated in discrete energy packets called quanta** and came up with a complex formula that seemed to solve the problem. It was the first formula that used Planck's constant h (= 6.63×10^{-34} Js). This was the start of the birth of modern physics.

The graph for Planck's findings looked like this (*Figure 27*):

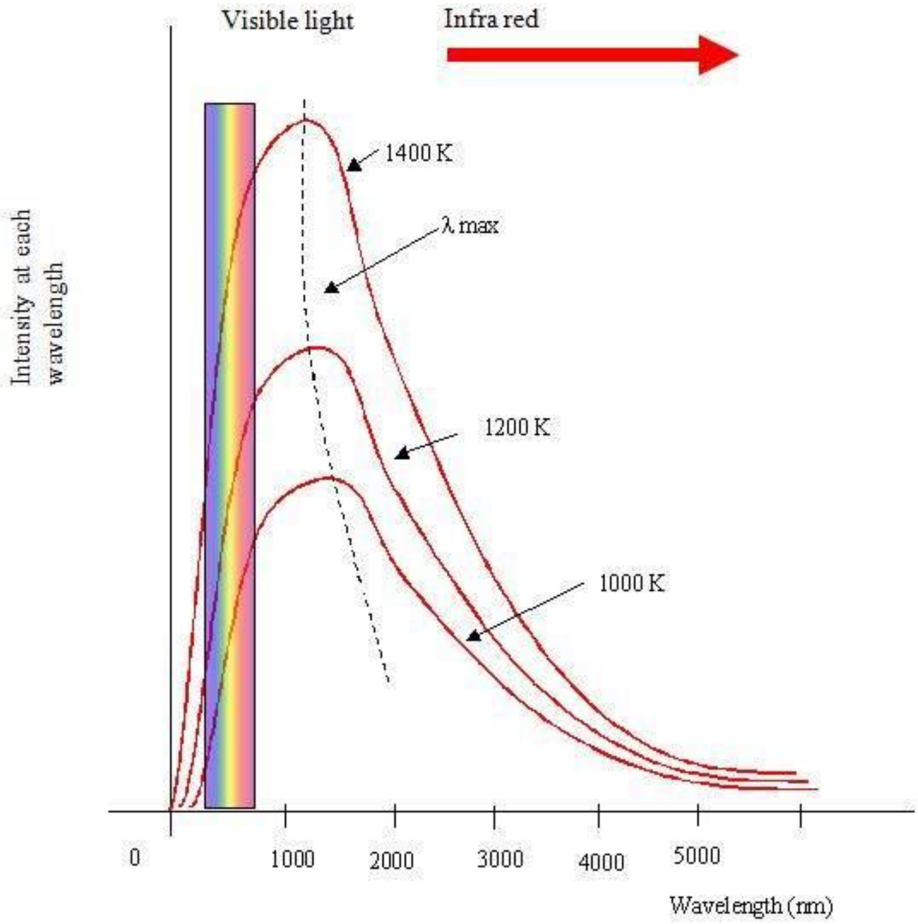


Figure 27 Planck's graph of photon intensity against wavelength

The graph is telling us that the **longer the wavelength**, the **lower the intensity**. Intensity represents the energy in a wave. So short wavelength waves carry more energy. We have seen that gamma rays are very energetic, for more so than infrared.

In 1905 Albert Einstein (1879 – 1955) extended the idea that when a quantum of energy is emitted by an atom, it continues to exist as a concentrated packet of energy. The energy of the packets (**photons**) was given by:

$$E = hf$$ .... *Equation 27*

14D.042 Photo-electric Effect

A beam of light was considered as a stream of particles:

- At long distances the intensity was low because the photons were spread apart.
- However, the energy of each photon was undiminished.

Einstein then went on to state that when a photon collides with an electron, it must

- either be reflected with no loss of energy,
- or must lose all its energy to the electron.
- The photon **interacted with one electron only**.

Therefore, the number of electrons being emitted was proportional to the number of photons that landed on the surface. Furthermore, electrons were emitted immediately the photons hit the surface. The effect can be shown with this experiment (*Figure 28*):

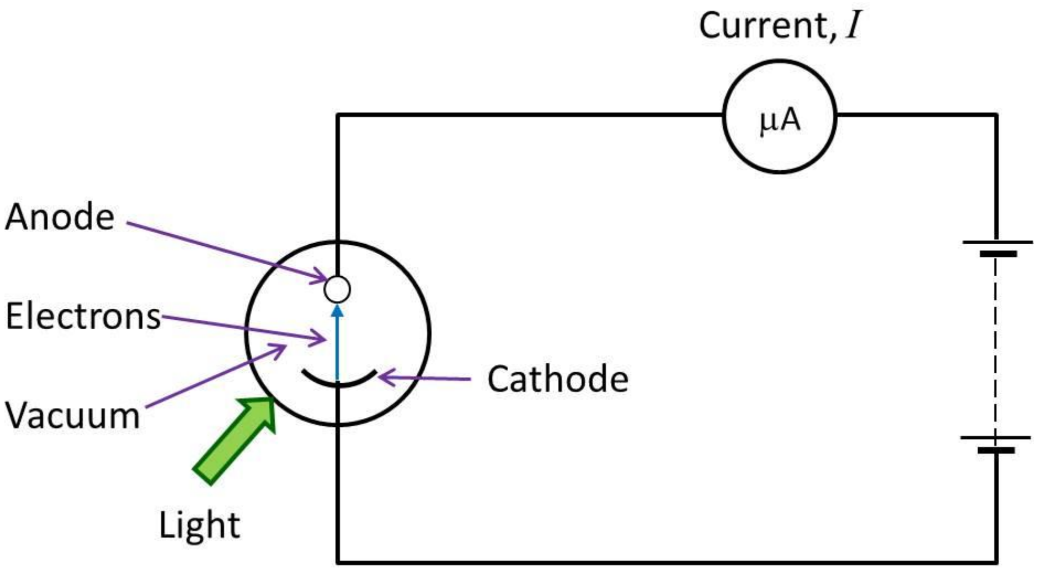


Figure 28 Demonstrating the photoelectric effect

The photocathode is made from a reactive metal such as caesium. A reactive metal loses its outermost electrons most easily.

The experiment showed that if light had a lower frequency (i.e. longer wavelength) than the threshold frequency, no current at all was observed, however bright the light was. Dim green or blue light would work, bright red light will not.

Using this apparatus we can measure the stopping voltage (*Figure 29*):

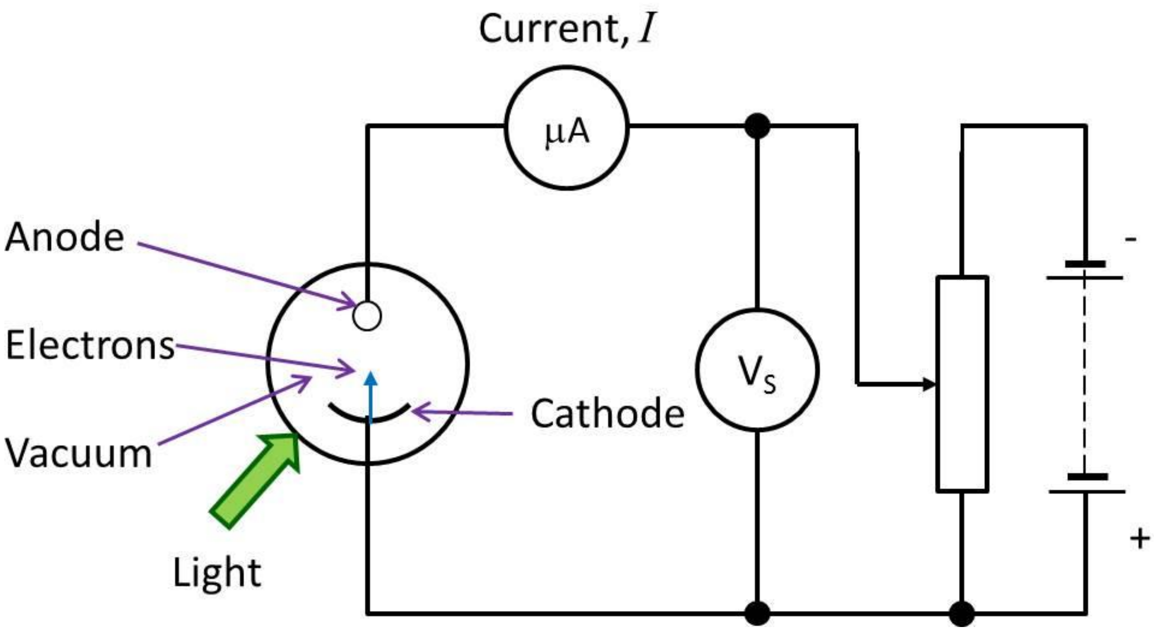


Figure 29 Measuring the stopping voltage

Note that in this experiment the **cathode** is connected to the **positive** terminal, and the **anode** is connected to the **negative**.

The two principal observations were:

- The maximum kinetic energy (indicated by the stopping voltage, V_s) depended on the frequency. The stopping voltage was higher for UV light than it was for green light.
- The more surprising observation was that no matter how bright or dim the light the stopping voltage was exactly the same.

Wave theory suggests that	***Supported or Contradicted?***
Any frequency can emit electrons	Contradicted!
Current depends on intensity	Supported
Maximum energy is independent of frequency	Contradicted!
Maximum energy would depend on intensity	Contradicted!

Einstein's explanation for these results was:

- Each photon provided energy for ONE electron to escape.
- Electron cannot escape if the energy in the photon is not sufficient.
- Photon energy $E = hf$.

Electrons would only have kinetic energy if they had been given sufficient energy to break the bond between them and their parent ions. The **threshold frequency** is the frequency of the photon that just has sufficient energy to break that bond. This energy is called the **work function** and is given the physics code Φ. The symbol Φ is "Phi", a Greek capital letter 'F'.

Photon energy = maximum kinetic energy + work function

In Physics code and rearranging:

$$hf = E_k + \Phi$$

.... *Equation 28*

Therefore:

$$E_k = hf - \Phi$$

.... *Equation 29*

Since all the energy in an electron is kinetic, we can work out the kinetic energy using the stopping voltage:

$$E_k = eV_s$$

.... *Equation 30*

We find out the stopping voltage by adjusting the potentiometer to the point where the current just reaches zero. At this point, the **negatively charged anode** is repelling the most energetic electrons, which are attracted back to the **positively charged cathode**. We are only interested in the most energetic photoelectrons. The rest are just also-rans.

So, we can now write:

$$eV_s = hf - \Phi$$

.... *Equation 31*

We can plot the data on a graph (*Figure 30*):

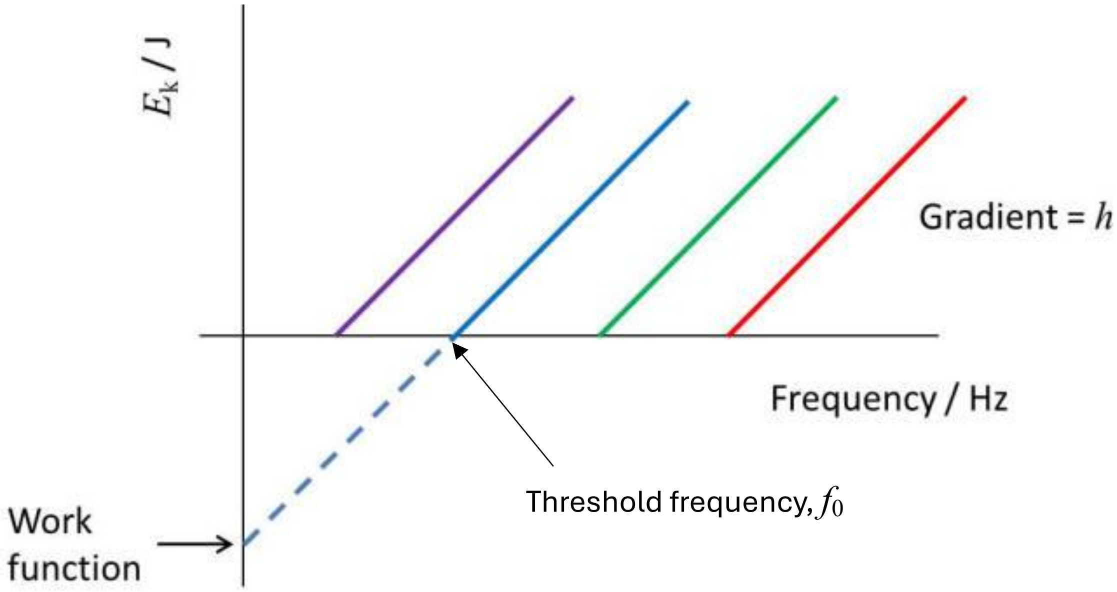


Figure 30 Photoelectron energy plotted against increasing wavelength

We can plot the kinetic energy for several different metals against the frequency. Note that the colours shown on the graph are there to illustrate different metals. They do not represent the colours of light. We find the following:

- The gradient is the same, whatever the metals.
- The gradient is always h, 6.63 × 10^{-34} J s.
- If voltage is on the vertical axis, the gradient is h/e = 4.14 × 10^{-15} J s C^{-1}.
- Each metal has a different value for its threshold frequency.
- The most reactive metals have the lowest threshold frequency.
- At the threshold frequency, $E_k = 0$.
- The intercept on the vertical axis gives the work function.

The experimental results agreed with Einstein's explanation. This experiment was good evidence that light was a particle. However, Young's slits showed that light was a wave. A conundrum...

Light travels as **a particle that shows wave behaviour**. It is emitted and absorbed as photons. This explanation is called **Wave-Particle Duality**.

14D.043 Enter the Count

Louis de Broglie (1892 – 1987), a French nobleman and historian, who had more than a penchant for Physics wrote a thesis in 1924 stating that if light waves showed particle properties, it was very reasonable to state that particles should show wave properties. Any particle of mass m would have an associated wavelength that could be worked out by:

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

.... *Equation 32*

You may wish to review this in Topic 3.

The wave behaviour of electrons, which are, of course, particles, can be shown with this experiment (*Figure 31*):

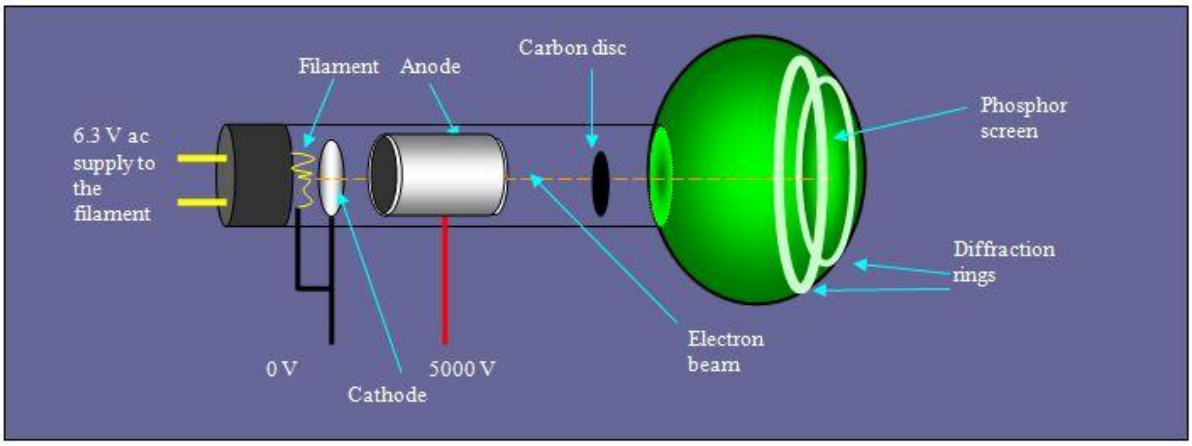


Figure 31 Electron diffraction

Electrons are diffracted at certain angles by a very thin layer of graphite to produce rings. The ring spacing fits very well the model predicted by the Bragg Equation:

$$\lambda = 2d \sin \theta$$ ………… *Equation 33*

This equation was worked out by the father and son team of William and Lawrence Bragg. It applied to diffraction of X-rays, which are, of course, electromagnetic waves. Therefore, we can say that electrons are showing **wave-like properties**.

We can find the momentum of the electrons easily enough:

Kinetic energy = electrical energy supplied = charge × voltage

$$\frac{1}{2}mv^2 = eV$$

………… *Equation 34*

Therefore:

$$mv^2 = 2eV$$

……….*Equation 35*

Now multiply both sides by m:

$$(mv)^2 = 2meV$$

………… *Equation 36*

If we square root this expression, we get:

$$p = mv = \sqrt{(2meV)}$$

…………… *Equation 37*

We can combine this with the **de Broglie relationship**:

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{(2meV)}}$$

.... Equation 38

A Young's slit type of experiment can be carried out with electron beams. And it is found that there is an interference effect. If we regard an electron as merely a particle, it is difficult to see it passing through two places at once. However quantum theory gives electrons **wave properties**, and there is no difficulty. Electrons are quantum beings that exist as probability. The closer you get to them, the less likely you are to catch the little brutes.

14D.044 Matter Waves

Electrons are considered to travel as **matter waves**, which are not composed of matter despite their name. It is instead to do with probability. Where the amplitude is greater, there is a greater probability of finding an electron. It is also true of photons. We can no longer think in quantum physics of particles like electrons being solid point masses like lead shots. Their **wave function** is extended over an extended region of space rather than being at a single point.

This is quantified by complex mathematical functions that are way beyond what we need to know. Werner Heisenberg was considering the matter waves of electrons, when he proposed his **uncertainty principle**. In effect, the closer you are to pinning down an electron the harder it is to pin down. (Heisenberg was a brilliant theoretical physicist, but useless at practical physics.)

These difficult concepts in quantum physics enable us to explain **tunnelling**, which we will look at next.

We know that an electron can have discrete energy levels (*Figure 32*).

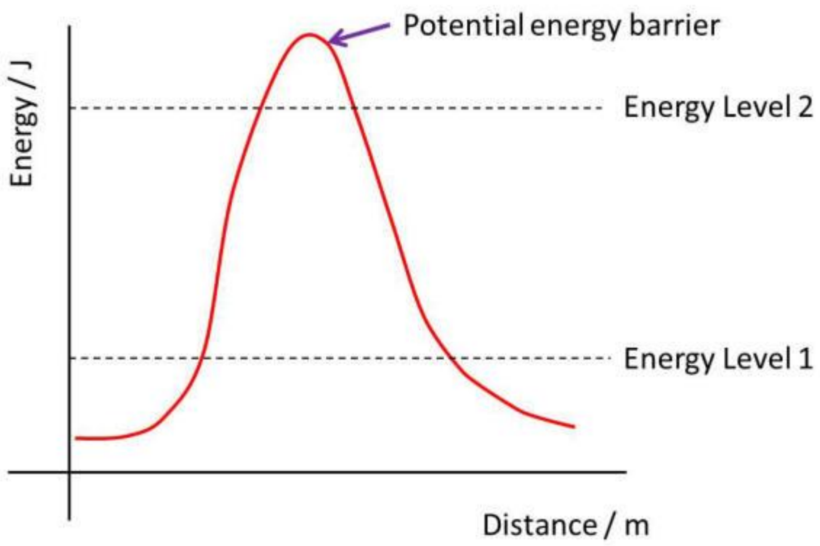


Figure 32 Potential energy barrier

According to **classical physics**, the electron can only be at energy level 1 or 2. It will not have enough energy to jump over the energy hill.

If we adopt the **quantum mechanics** idea, where the bigger the amplitude of a matter wave, the greater the probability of finding the electron, we get this (*Figure 33*):

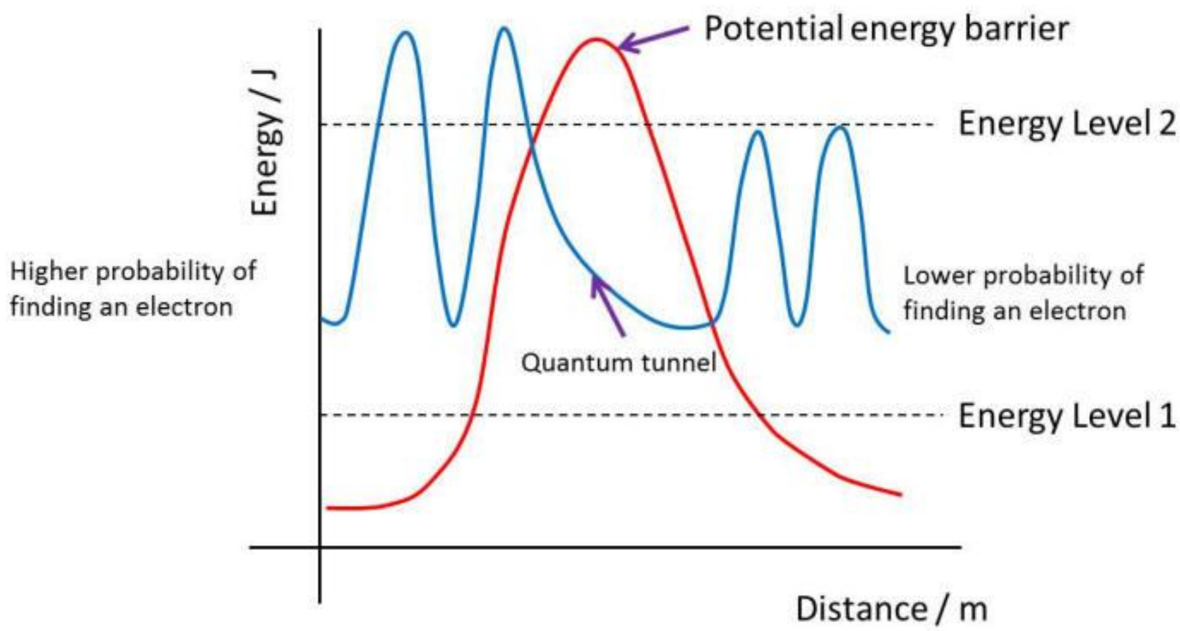


Figure 33 Quantum tunnelling

There is a small but finite probability of finding the electron on the other side of the energy hill. It seems to have tunnelled through the energy hill, and this effect is called **quantum tunnelling**. It is impossible to explain this by classical physics, but (comparatively) easy to use quantum physics which is based on probability.

Let's use a simple analogy. "*Fingers*" is a prisoner in the nick. He doesn't want to do any more porridge, so he wants to get out. "*Fingers*" is in the prison's exercise yard. 10 metres to his left is the outside. But he has to get over the wall, which will be noticed. Besides, most prison walls are capped with smooth cylindrical structure to prevent escaping prisoners from getting a handgrip or using grappling irons. In **Classical Physics**, he needs to jump over the prison wall, but he cannot jump high enough (*Figure 34*).

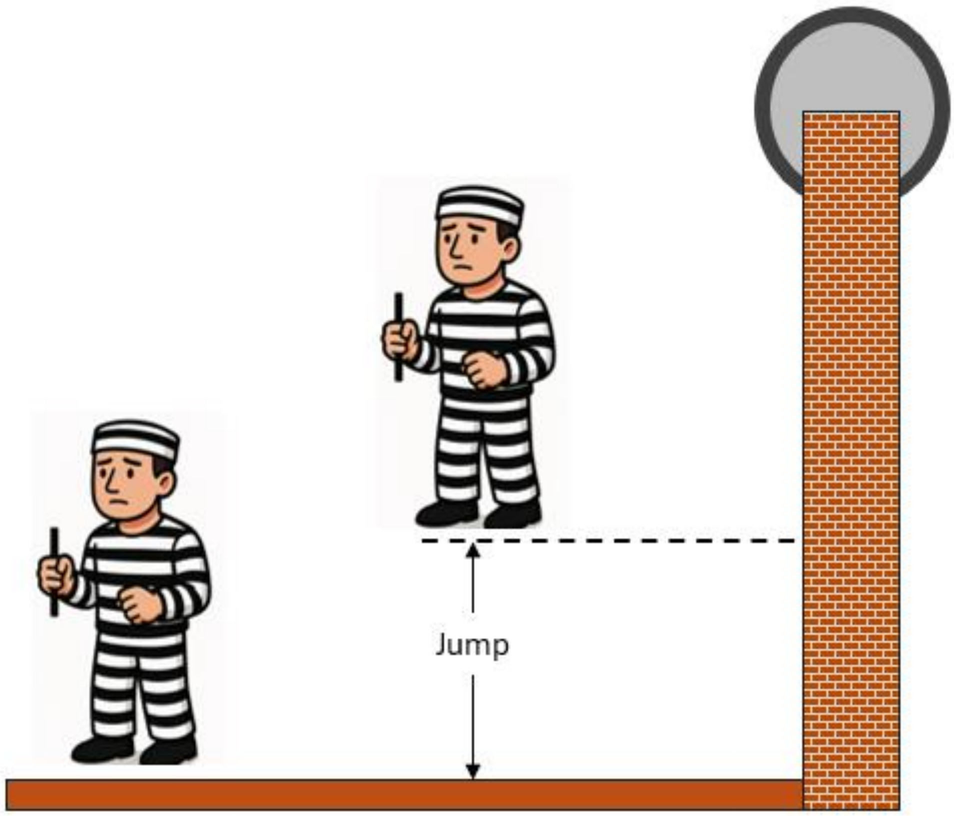


Figure 34 Fingers cannot jump over the wall to escape

In the **quantum world**, electrons do not orbit the nucleus like satellites orbiting a planet as shown in most school textbooks. Instead, electrons move around a probability cloud, jumping from one discrete energy level to another. While for most of the time, an electron is at lower energy levels, there is a probability that an electron can have sufficient energy to cross the potential barrier. It's as if "*Fingers*" can exist in a probability cloud where he is in an energy level that allow him to cross the prison wall (*Figure 35*).

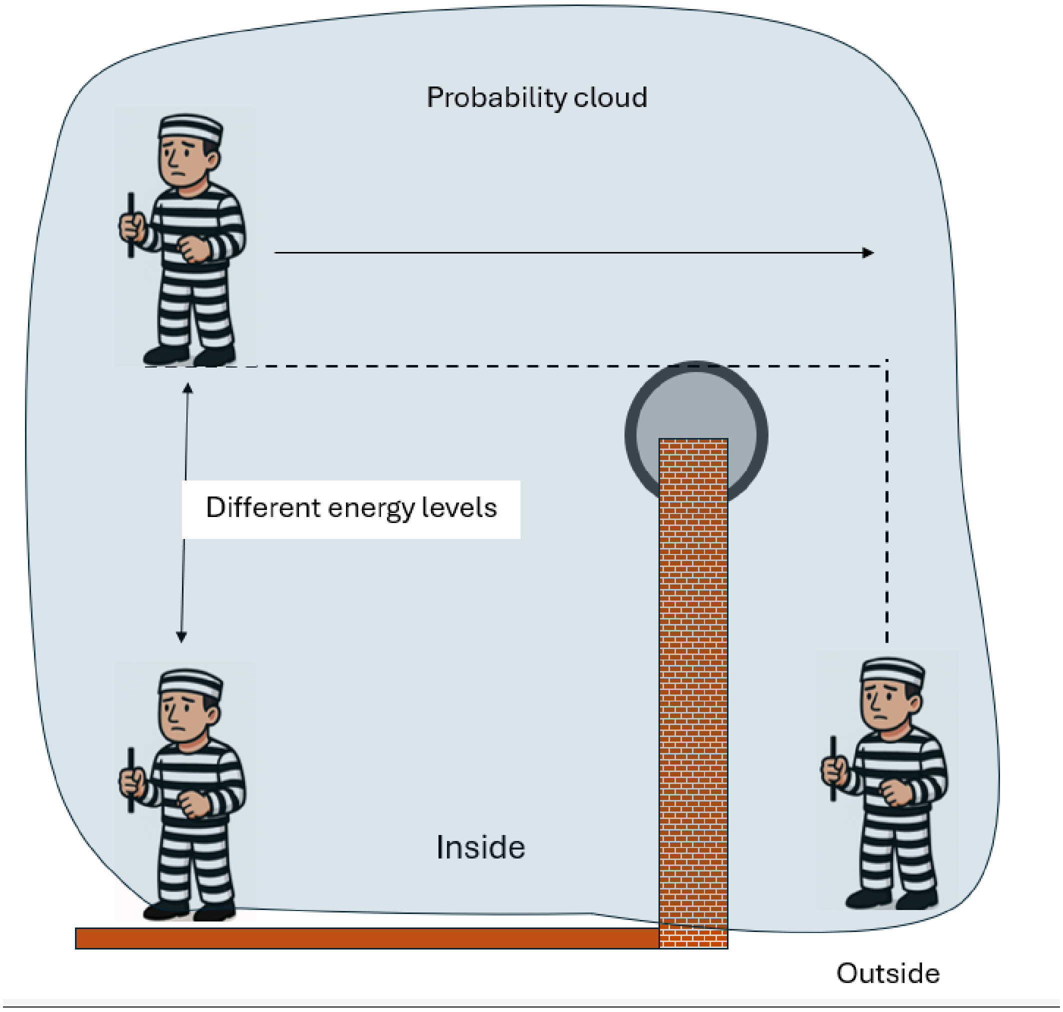


Figure 35 Fingers has made his escape

In *Figure 35*, there is a very small, but definite probability that "*Fingers*" can achieve an energy level that allows him to cross the wall and drop down again to the outside. It is as if he has tunnelled through the wall.

Electrons exist in a probability cloud. There is a small, but definite probability that they can cross a potential barrier. It is as if they have tunnelled through the barrier. This process is called **quantum tunnelling**. We will look an example this in the next section 14D.056.

Quantum Physics can use very complex mathematical models that are far beyond what we need to know here.

14D.055 Electron Microscopy

The light microscope can resolve objects of size about 0.5 mm, which is about 1 wavelength of visible light. If we consider accelerated electrons to have a de Broglie wavelength of about 10^{-10} m, we can resolve down to about the size of an atom.

The **transmission electron microscope** uses electrons in the same way as a light microscope uses photons. The general layout is like this (*Figure 36*):

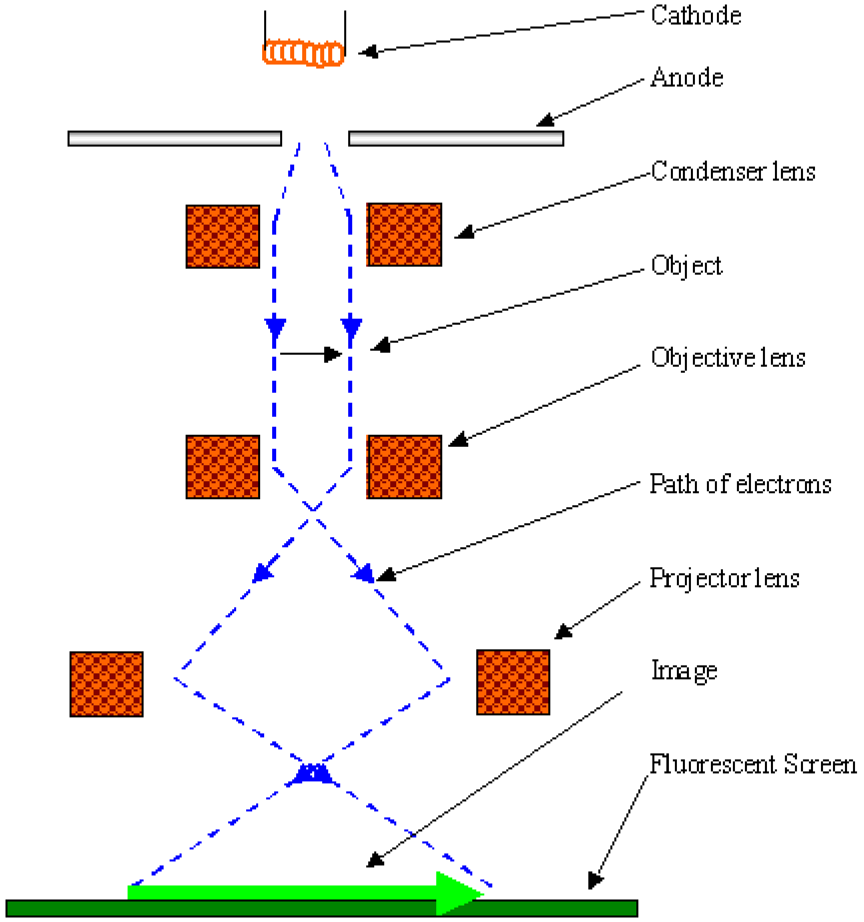


Figure 36 The transmission electron microscope

The transmission electron microscope has the following features:

- Electrons accelerated through a pd of 100 kV have a wavelength of 10^{-12} m.
- Electrons are focussed by magnets that act as lenses. In the diagram the lenses are electromagnets.
- The electron microscope can resolve much finer detail as the wavelengths are 10 000 times smaller.
- Increasing the anode voltage gives better resolution (finer detail).

Problems:

- Electrons lose momentum going through the sample. This makes their wavelength increase, reducing detail.
- There is a range of electron speeds, hence wavelengths. This can distort the image, rather like chromatic aberration in a light microscope.
- The focal length of a magnetic lens depends on the current. In practice there are small variations which leads to slight changes in the focal length.
- The electron path must be in a high vacuum to prevent collisions with electrons and gas molecules.

Although the electron microscope can in theory resolve down to about 2×10^{-12} m, in practice the resolution is about 2×10^{-10} m. Even so this has allowed:

- Physicists to resolve individual atoms.
- Biologists to study the ultra-structure of the cell and how viruses are made up. Viruses are far too small to be seen with the light microscope.

14D.056 The Scanning Tunnelling Electron Microscope

It was invented in 1981 by Binnig and Rohrer to provide extremely high resolution images of the surfaces of materials. It relies on the quantum physics effect of tunnelling which we looked at earlier (14D.054).

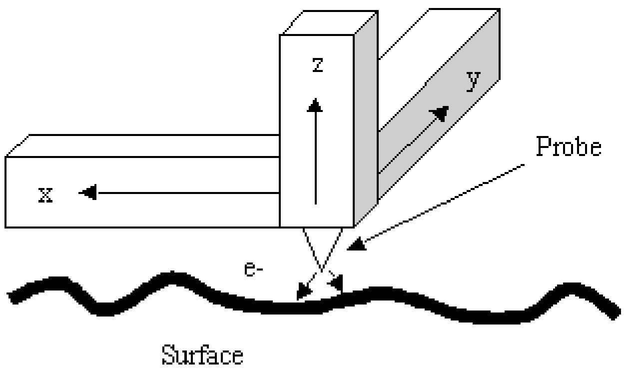


Figure 37 Quantum tunnelling electron microscope

The probe has an extremely sharp tip which is kept at a small negative voltage compared to the surface (*Figure 37*). The surface must be able to conduct electricity; insulating materials must be coated with a thin conducting layer. The probe is moved in the x, y, or z coordinates by quartz crystals which change shape when a small current is applied. (This is the reverse of the **piezo-electric** effect which you may have come across in some gas lighters. The quartz crystal is squeezed and gives out a high enough voltage to give a spark.) The probe scans across the surface like this (*Figure 38*):

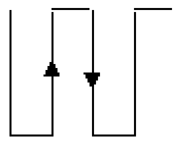


Figure 38 Scanning a surface using the quantum tunnelling electron microscope

- If the gap is less than 10^{-10} m some electrons may cross the gap producing a tiny current.
- As the gap decreases, the current increases. Changes as little as 10^{-12} m can be detected.
- The height of the probe is continually adjusted to keep the current constant. In this way the probe traces out the profile of the surface.
- The *x*, *y*, and *z* coordinates are controlled by a computer.
- The image generated is displayed on a VDU.

Extremely fine images of atoms in crystals can be generated.

Questions

Tutorial 14D.04

14D.04.1

Look at the introductory paragraph on Page 41. If the wave model of light were correct, what would you expect to see with bright red light?

14D.04.2

Explain what each of the terms in this equation stand for and give the units:

E:

h:

f:

14D.04.3

What do you think will happen to the current if the intensity of light is increased. Does this contradict the wave theory?

14D,04.4

Gold has a work function of 4.9 eV.

(a) What is this in joules?

(b) What is the maximum kinetic energy that the photoelectrons have if the gold is illuminated by UV light of frequency 1.7×10^{15} Hz

(c) What is the stopping voltage of these electrons?

(d) How fast do the electrons travel as they leave the surface?

Data:

$h = 6.63 \times 10^{-34}$ Js

$e = 1.6 \times 10^{-19}$ C

mass of electron = 9.11×10^{-31} kg

14D.04.5

What is the de Broglie wavelength of an electron that has been accelerated from rest by a p.d. of 40 V?

14D.04.6

What is the wavelength of 100 keV electrons?

14D.04.7

Write down and explain two problems that limit the resolution of the electron microscope.

3. Relativity

Tutorial 14D.05 Relativity and the Speed of Light

AQA Syllabus

Contents

Relativity was used initially to help physicists in proving the idea of **luminiferous ether**, a mysterious substance in which light waves had to travel. The thoughts in those days was that all waves had to travel in a material. This was at a time when it had been conclusively demonstrated that light was a wave. The idea of photons had yet to catch on.

We will look at relativity and its use in trying to prove the existence of ether, and the way that the speed of light was measured. As well as his experiment to show ether, Albert Michelson demonstrated the speed of light, the value of which is accepted today.

14D.051 Basic Relativity

When we measure movement, we do so against a **fixed reference point**. A car travelling at 30 m s^{-1} is moving at 30 m s^{-1} relative to the road. Suppose we have two cars, A travelling at 30 m s^{-1} and B travelling at 20 m s^{-1} (*Figure 39*).

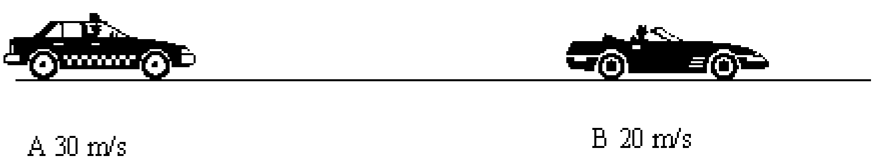


Figure 39 Relativity with two cars

- Relative to B, car A is travelling 10 m s^{-1} faster, i.e. +10 m s^{-1}.
- Relative to A, car B is travelling 10 m s^{-1} slower, i.e. –10 m s^{-1}.

We can use any of these **frames of reference**:

- The road.
- Car A.
- Car B.

Another example is an aeroplane flying at 90° to the wind (*Figure 40*):

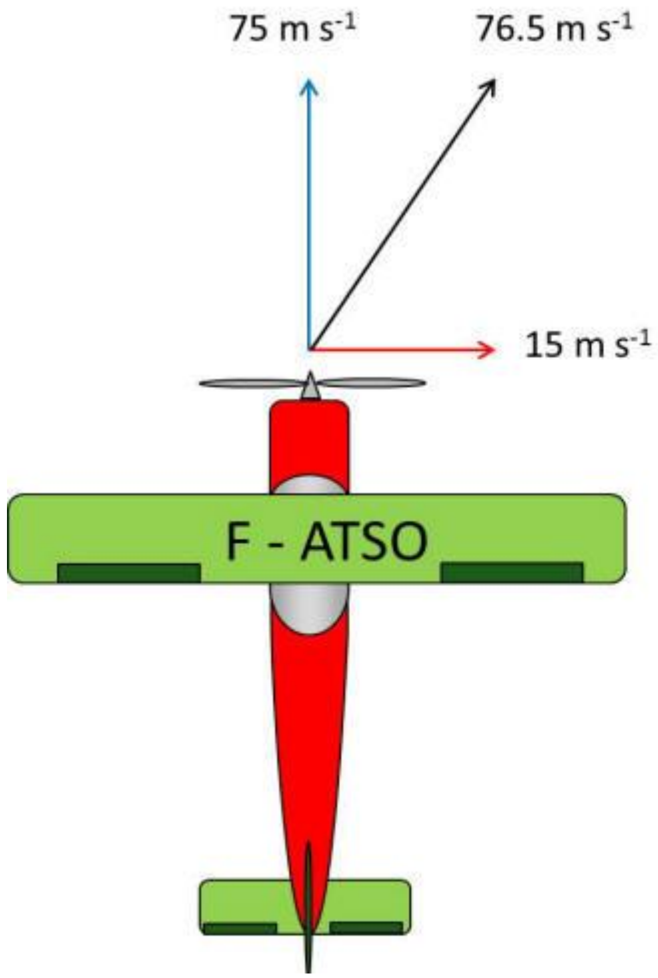


Figure 40 This plane is flying into a crosswind

The plane is heading due North at 75 m s^{-1} and the wind is blowing from West to East at 15 m s^{-1}. We can easily work out the resultant velocity to be 76.5 m s^{-1}. There can be three frames of reference on the ground:

- The speed is 75 m s^{-1} heading due North.
- Or 15 m s^{-1} due East
- Or a resultant velocity of 76.5 m s^{-1} at 11.5° east of north.

The question that bothered physicists was whether there was an **absolute fixed point** relative to which all speeds could be measured.

14D.052 Galilean Relativity

Galileo Galilei (1564 - 1642) was one of the first people to carry out experiments on the **motion of objects** (which we now call **mechanics**). He also carried out the first **thought experiments** and one of these forms the basis of understanding frames of reference. Would a mechanics experiment behave in a different way depending on whether it was stationary or moving?

Consider a ship which is stationary. A stone is resting on a platform on the mast of the ship (it's a ship of Galileo's era) like this (*Figure 41*). OK, The ship here is a modern replica.

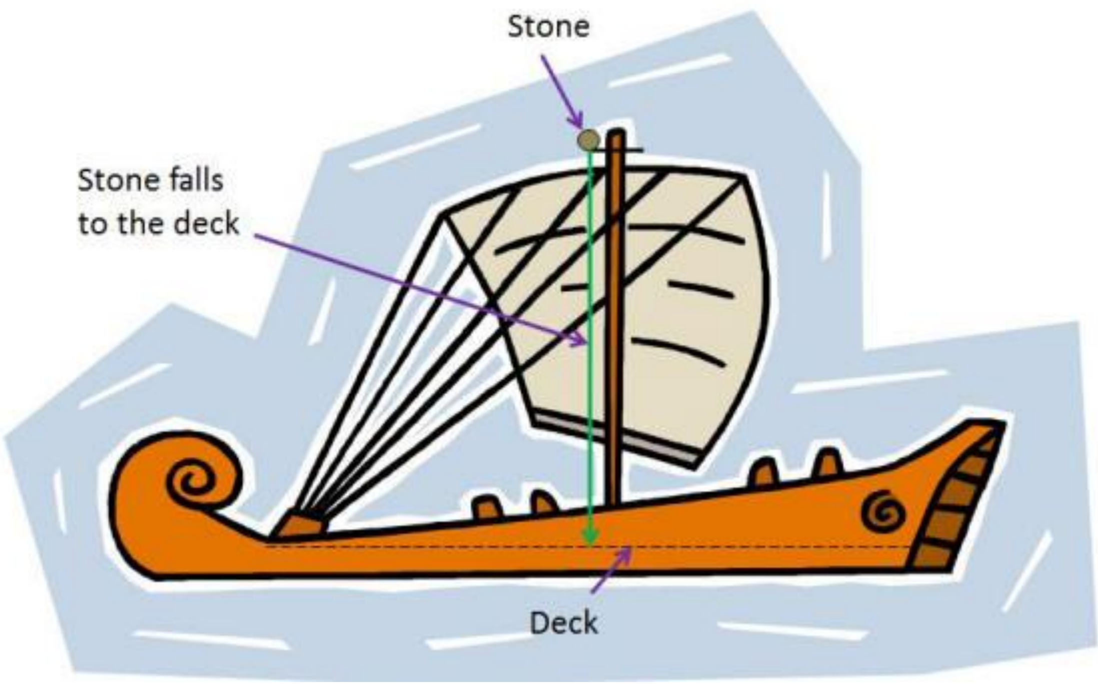


Figure 41 Dropping a stone from mast of a ship

We would be right in expecting that the stone would fall vertically to the deck.

Now suppose the ship is moving at 5 m s^{-1} along a perfectly flat sea (*Figure 42*). It does not rock from side to side (roll) or pitch forwards or backwards. What happens if we drop the stone from the platform now? We will assume that the time taken for the stone to fall is 1.0 s, and this gives a mast height of 4.9 m (how?).

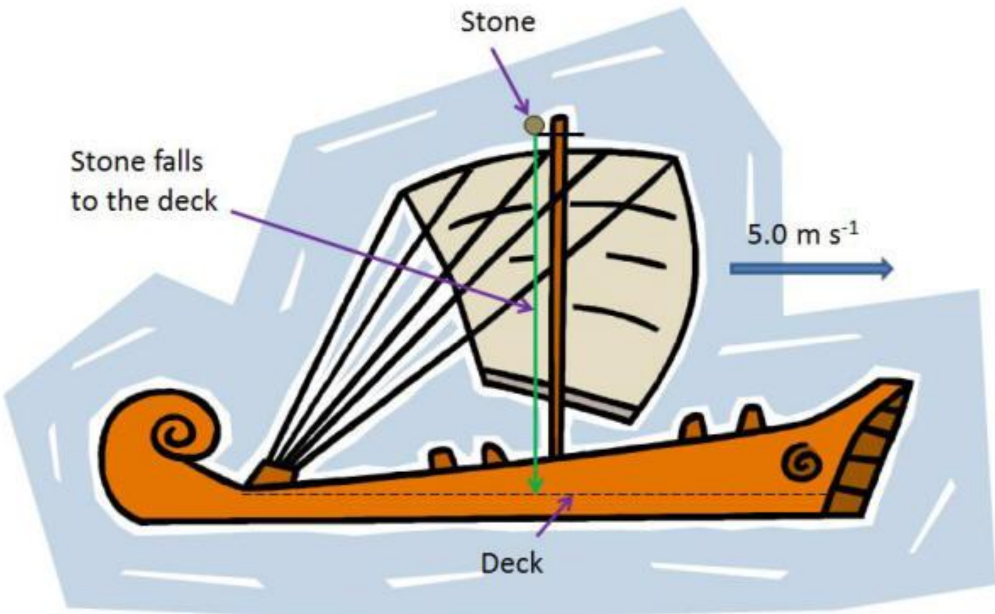


Figure 42 Stone falling on a moving ship

Does the stone hit the deck where it did before, or does it hit the deck 5.0 m behind? The answer is of course that the stone hits the deck in exactly the same place as before. This is because the stone is moving at 5.0 m s^{-1} along with the rest of the ship. The idea is shown below (*Figure 43*):

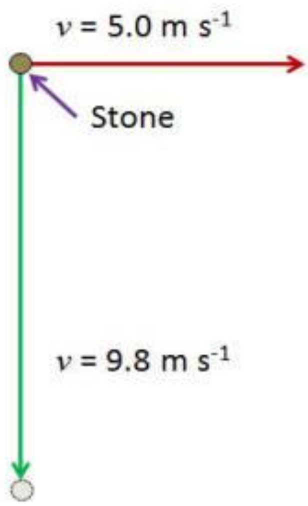


Figure 43 Vector diagram of the stone falling

If you are on board the ship, you would not be able to tell the difference in the outcome. However, if you are watching the stone falling while you are standing on the quayside, you will see this (*Figure 44*):

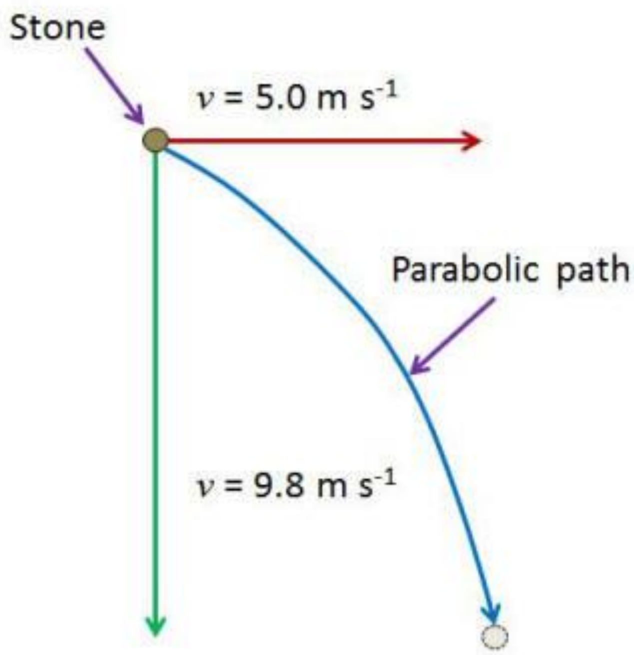


Figure 44 Vector diagram of the stone falling as seen outside of the ship

The stone would move on a **parabolic** path. If you want to review the behaviour of the stone as it falls, go to Topic 5 in order to revise projectile motion.

The key point is that there is no difference in outcome between a mechanics experiment being carried out in a stationary environment and one carried out in an environment that is moving at a constant velocity with zero acceleration. (As soon as there is acceleration, the outcome will be different.) We say that the ship forms one **frame of reference**, while the quayside is a **second** frame of reference. In each frame of reference, there is an **observer** to see what happens. The observer on the ship will see the stone falling in a vertical path, while the observer on the quayside will see the stone falling in a parabolic path.

We call this principle **Galilean Relativity**.

When something moves, we can describe its movement in terms of the x-axis (horizontal axis across the page), the y-axis (horizontal axis into the page) and the z-axis (upwards). Let's say that the ship is travelling at v m s^{-1}. In a time t seconds, it will travel vt metres. For the observer on the quayside, the position of the ship can be described as a set of **coordinates**:

$$(x, y, z, t)$$

For the observer on the ship, the coordinates will be:

$$(x', y', z', t')$$

(x' is pronounced "ex-prime" or "ex-dashed" or "ex-stroke". It depends on your tutor.)

However, in this case, we will keep things simple and consider the movement solely in terms of the *x*-axis, i.e., 1 dimension. The equations that relate the two are called Galilean transforms and are listed below:

$$x' = x + vt$$ .... *Equation 39*

$$y' = y$$ .... *Equation 40*

$$z' = z$$ .... *Equation 41*

$$t' = t$$ .... *Equation 42*

In this case, y remains the same, as is z, as is t. **Time** t does not vary at all in Galilean Relativity. It is called a **physical invariant**.

Let's suppose that the mast is at a point 15 m from the stern, as shown in *Figure 45*:

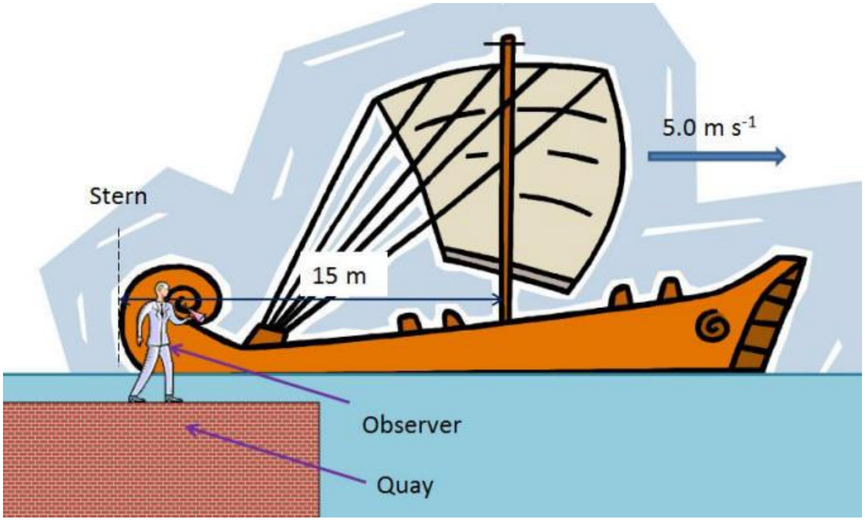


Figure 45 The ship observed from the quayside

For an observer at the stern of the ship, the mast will not move. It will remain 15 m in front of the observer. For the observer on the quay, let's say that at time 0, the stern passes the observer. The mast is 15 m in front. If we then plot the position every second:

$$x'_0 = 15\text{ m} + 5.0\text{ m s}^{-1} \times 0\text{ s} = 15\text{ m}$$

$$x'_1 = 15\text{ m} + 5.0\text{ m s}^{-1} \times 1\text{ s} = 20\text{ m}$$

$$x'_2 = 15\text{ m} + 5.0\text{ m s}^{-1} \times 2\text{ s} = 25\text{ m}$$

and so on...

We can plot a distance time graph like this (*Figure 46*):

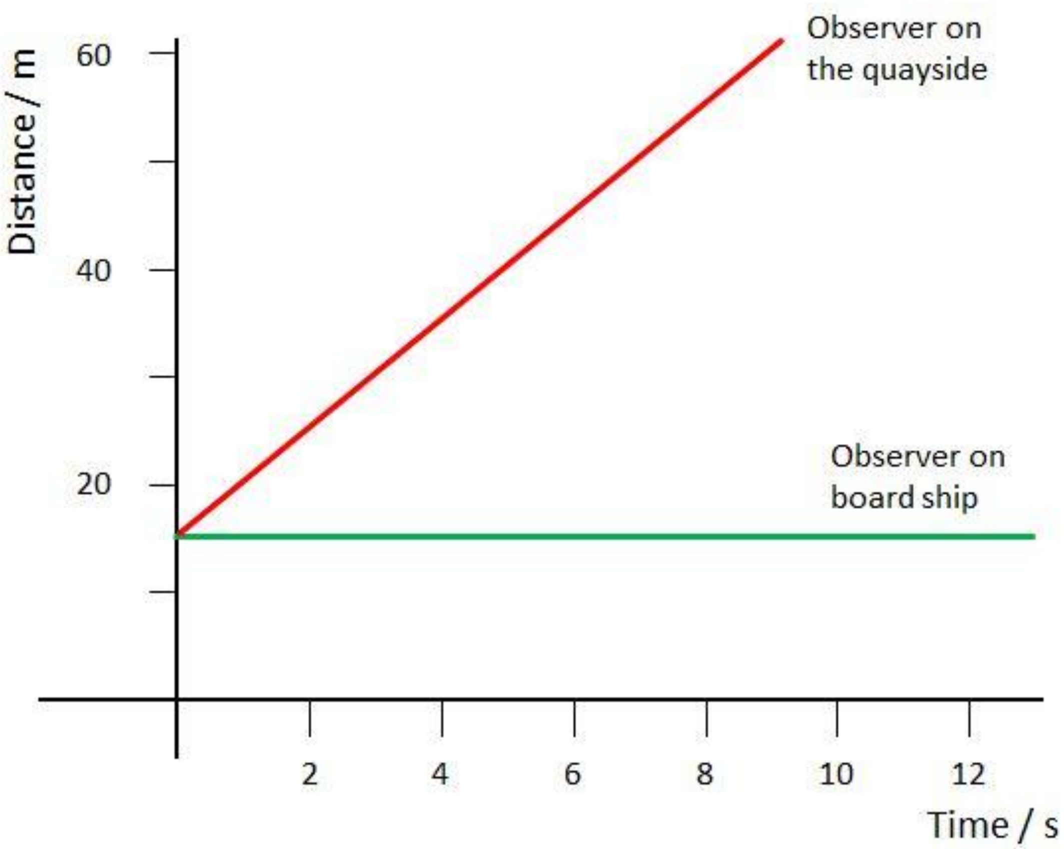


Figure 46 Distance-time graph for the ship

A more modern context would be a juggler playing on an aeroplane travelling at a speed of 200 m s^{-1} at a steady altitude of 10 000 m (*Figure 47*).

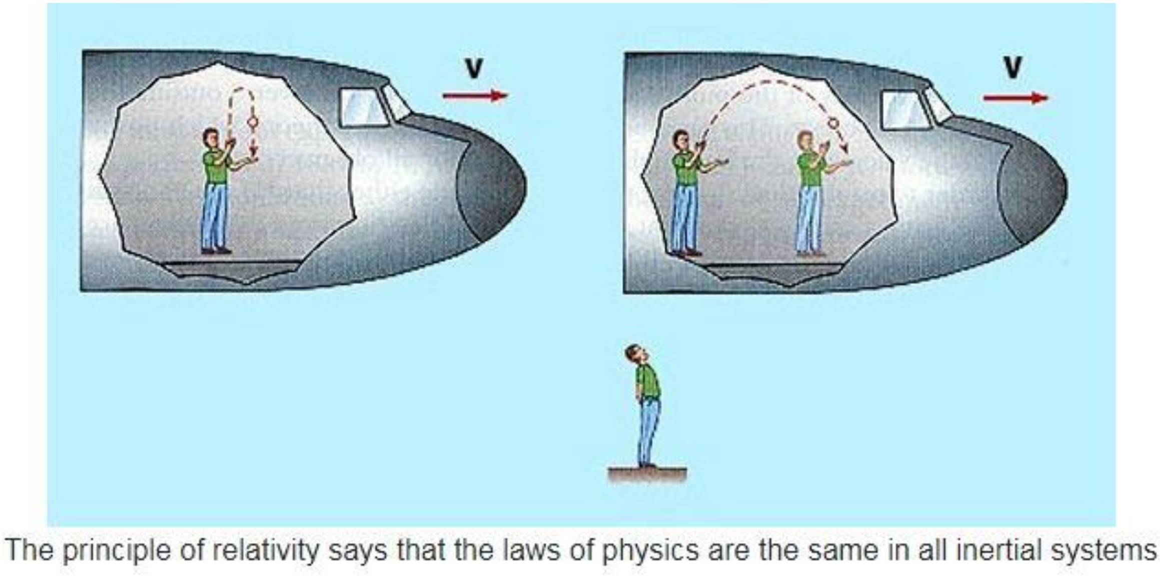


Figure 47 A more modern version of the Principle of Relativity

Source: https://www.physicsoftheuniverse.com/topics_relativity_light.html

14D.053 Newton's Postulates

Among the many contributions that Isaac Newton has made to our understanding of physics are his two **postulates**. (A postulate is a thing suggested or assumed as true as the basis for reasoning, discussion, or belief.)

The first one is that there is **absolute time**. This is **independent** of any observer. We cannot observe absolute time, only the **passage** of time, which we do using clocks, days, months, seasons, etc.

Perhaps a way of thinking about this is to imagine getting on a train at an intermediate station. We know that the train started somewhere, but we don't know the exact time it left (yes, we can use the timetable, but we cannot be sure that it left on the dot). When we get off the train at another intermediate station, we know that the train will go on to its terminal station, but we don't give a damn whether it arrives on time there. All we are interested in is the time we are on the train.

Absolute time, according to Newton, was something that could only be understood mathematically.

Newton's second postulate was that **space** was **absolute**. It formed a backdrop to events. Every object has an absolute state of motion. For example, the Earth is orbiting the Sun in a constant motion at a speed of 30 000 m s^{-1}. The Sun is moving as well. As I sit here typing this, listening to *Radio 3*, I am stationary, as is my house and my garden. My surroundings are stationary in a **relative** sense. However, they are moving in an absolute sense.

Absolute motion is the movement of objects in **absolute space** from one **absolute position** to another. **Relative motion** is the motion of objects from one **relative position** to another relative position.

14D.054 A Boat Race

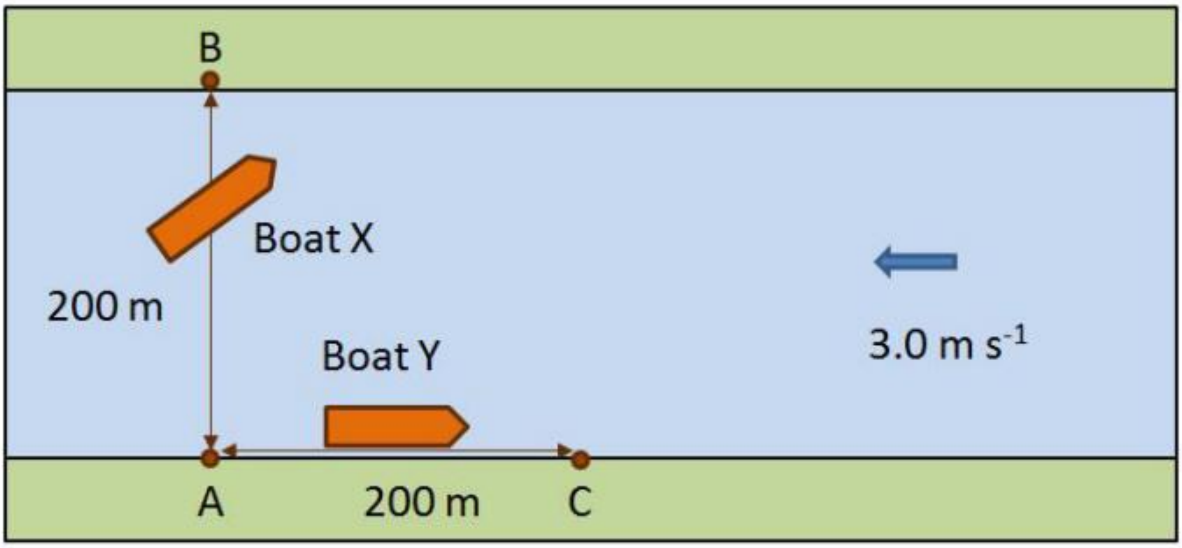


Figure 48 A boat race

Both boats, X and Y have a speed of 5 m s^{-1}. Boat X has to cross the river from A to B and back to A, while Boat Y has to travel from A to C and back again. Have a go at Question 14D.05.1.

14D.055 Inertial Frame of Reference

Answer Question 14D.05.3.

An **inertial frame of reference** is one in which Newton I is valid. If you are in a train travelling at constant speed, all objects behave as if they were stationary in the stationary train. The train is travelling at 60 m s^{-1}, the passengers and their luggage are all travelling at 60 m s^{-1}.

Suppose now that you are in an aeroplane. Against all airline regulations, there is a drinks trolley free (not secured) in the central aisle. The aeroplane accelerates down the runway. From within the plane the trolley appears to accelerate towards the back of the plane.

- From the ground, the trolley obeys Newton I since there is zero force acting on it, hence zero movement.
- From within the plane, an **accelerating** frame of reference, the trolley appears to accelerate, which is not consistent with Newton I.

Answer Question 14D.05.3.

Now consider this situation. A person is standing at the centre of a roundabout that can revolve. He has a gun. A target is placed outside the roundabout as shown (*Figure 49*):

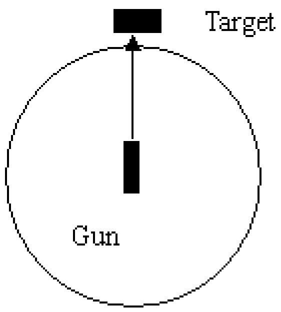


Figure 49 A gun fired at a target when the roundabout is stationary

When the roundabout is stationary, it is easy to see that the path of the bullet is straight. What about when the roundabout is turning (*Figure 50*)?

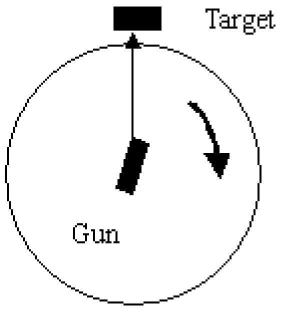


Figure 50 The roundabout is turning

For an observer on the ground the path of the bullet will be a straight line. For the person on the roundabout, the path will appear curved (*Figure 51*).

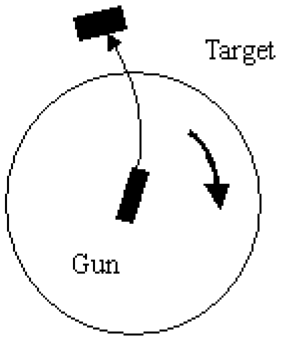


Figure 51 Path of the bullet from the point of view of the person on the roundabout

Answer Questions 14D.05.4 and 14D.05.5

14D.056 The Michelson-Morley Experiment

In order to explain wave phenomena such as light waves, the late nineteenth century physicists depended on a medium called **ether**. (It has nothing to do with diethyl ether; an explosively flammable compound used in organic chemistry.) Ether was a mass-less and non-viscous material that was needed to carry waves. Ether is used nowadays as a poetical word to describe radiobroadcasting.

If ether permeated the whole of space, then it would provide a perfect frame of reference to determine absolute motion. The experiment was carried out in 1887 by Albert Abraham Michelson (1852 – 1931) and Edward Williams Morley (1838 – 1923). Their idea was to measure the speed of light parallel to the Earth's motion with the speed of light perpendicular to it. It would be rather like the boat race example we saw above.

They used the physics of **optical interference** in a set up like this (*Figure 52*):

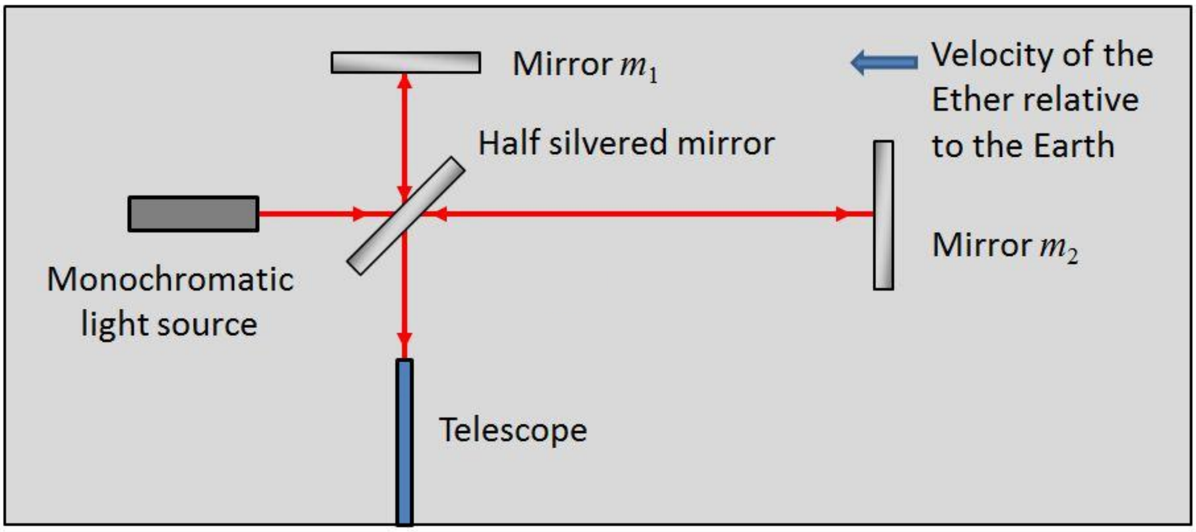


Figure 52 The Michelson-Morley experiment

- Light is split into two perpendicular beams.
- They travel to the mirrors and superpose as they return to give interference fringes.
- If the distance between the half-silvered mirror and m_1 is the same as the distance between the half silvered mirror and m_2, the time taken would be different.
- This would indicate a shift in the expected interference pattern.
- The experiment was repeated with the equipment set at 90° to the orientation of the first experiment, so that the motion in the ether would be observed in two different directions.

- It was repeated at different times of the year in case the sun at one point or another was moving in the same frame of reference.

Although the Earth orbits about the Sun, and is technically an accelerating frame of reference, we can generally treat it as an inertial frame of reference.

The results were the most important **null** (nothing) **result** of the time:

- There was **no difference in the speed of light** whichever way the experiment was done, or whatever the time of year.
- There was **no absolute reference point**.
- There was **no such thing as ether**.

Michelson never gave up on the idea of ether. He tried again and again but never found it.

Einstein's Theory of Special Relativity suggested that ether could not exist, and that the speed of light is a universal constant.

14D.057 Experiments to Measure the Speed of Light

The measurement of the speed of light foxed the early physicists. Galileo tried it, but his results were inconclusive. It is not easy. Many thought that the speed of light was infinite. The first scientist to give an estimate of the speed of light was Ole Römer (1644 - 1710), a Danish astronomer. From astronomical observations of one of Jupiter's many moons, Io, he came up with a figure of about 3.1×10^8 m s^{-1}.

The way that the speed of light can be measured is to chop it up into very small pulses. This can be achieved with rotating mirrors or with a toothed wheel. The apparatus below was devised by Hippolyte Fizeau (1819 - 1896). See *Figure 53*.

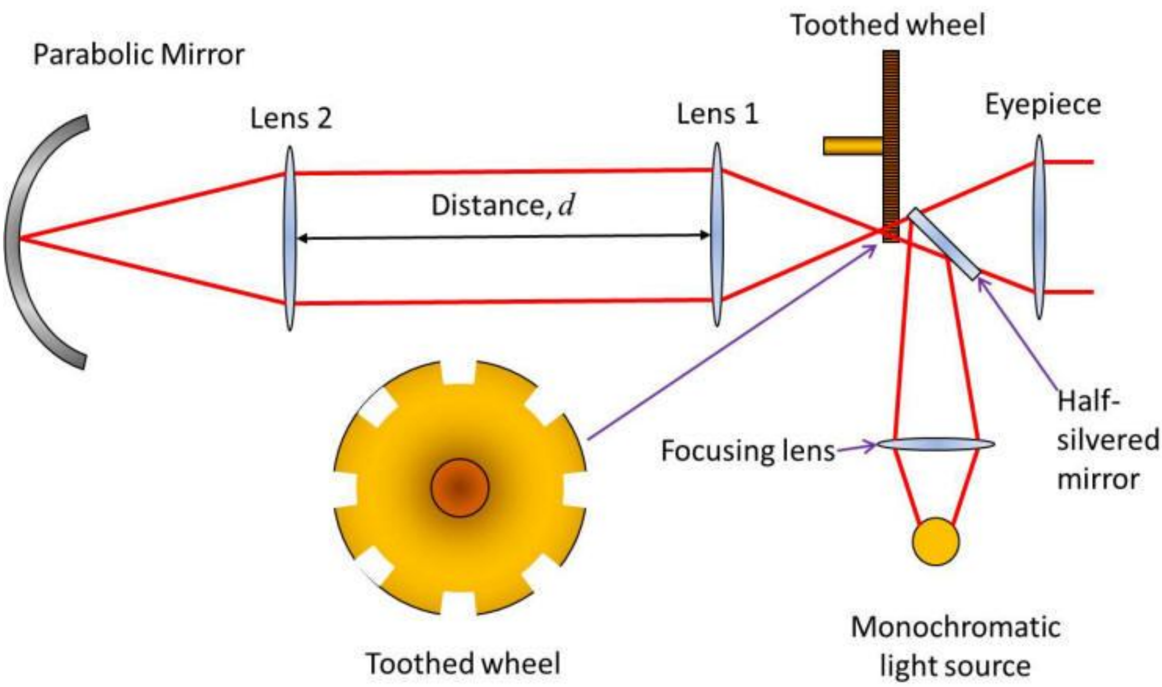


Figure 53 Fizeau's experiment to measure the speed of light

Light from a bright monochromatic light source is focused onto a focal point. There is a half-silvered mirror to bend the light through 90 degrees onto the focal point. The light is picked up by Lens 1, which makes parallel rays to travel a distance d to Lens 2. The rays reflected by a parabolic mirror back to Lens 2 and Lens 1. The light passes through the focal point and passes through the half-silvered mirror to the eyepiece.

The toothed wheel is spinning. The idea is that the ray going out is allowed to pass through the gap. The ray goes to the mirror and returns. In the meantime, the wheel has turned a small amount, but enough to block off the returning ray (*Figure 54*):

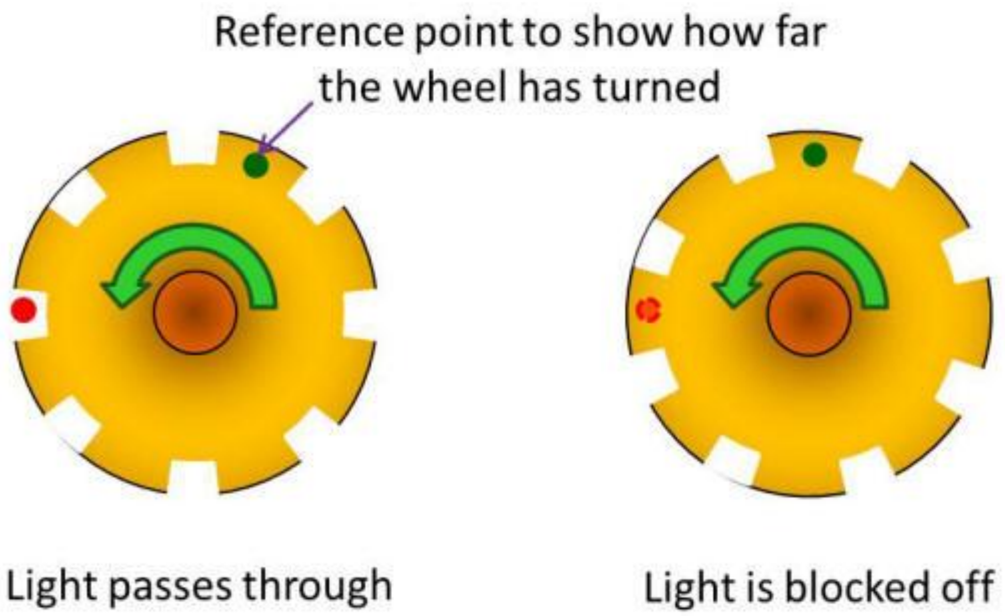


Figure 54 Measuring how far the wheel turns in the time taken for a pulse of light to travel.

Therefore, no light is observed at the eyepiece. The first time this is observed would be when the next tooth replaces the gap and blocks off the light that has returned.

The light travels from the transmitter to the reflector, a distance d (which is quite large, several kilometres). It then has to travel back again, covering a total distance of $2d$. To work out the speed, we also need to know the time. That is not so easy but can be worked out if we know how fast the toothed wheel is spinning.

Suppose the wheel, which has N teeth, is turning at n revolutions per second. Remember that for N teeth, there are N gaps. In the picture above, there are 8 teeth, and 8 gaps. The time taken for the light to get out and back is given by:

$$t = \frac{1}{2Nn}$$

.... *Equation 43*

Since speed = distance ÷ time, we can write:

$$c = 2d \div \frac{1}{2Nn} = 4dNn$$

...............*Equation 44*

Using these data:

- d = 8.63 km.
- N = 720 teeth.
- n = 12.6 revolutions per second.

we can now work out the speed of light using *Equation 44*:

c = 4 × 8.63 × 10^3 m × 720 × 12.6 s^{-1}

= **3.13 × 10^8 m s^{-1}**

There were some difficulties. The toothed wheel was made by a clock maker, with 720 teeth and 720 gaps. It was not easy to make. Measuring long distances could be done by triangulation with good precision. The main uncertainty would be determining the precise rate of turning of the toothed wheel. Since the experiment was done in 1849, the technology was not as sophisticated as today. So, a reliable measure of the rate of rotation was less likely. Also, the toothed wheel was spun, not with an electric motor, but with a **clockwork mechanism**. Also, the light source would not be that bright - lasers were not invented. There would also have been problems with the scattering of light as it passed through the air. The experiment would need to be done on a clear day (or night).

The experiment can be reproduced nowadays with a toothed wheel driven by an electric motor. An electronic tachometer can give an accurate read out of the speed.

A similar experiment was carried out by Leon Foucault (1819 - 1868), but with a spinning mirror.

In 1926 Michelson gave the first accurate measurement of the speed of light using a system of concave mirrors and a rotating octagonal mirror. The parabolic mirror on the left was set up on a mountain that was about 36 km from the base station (*Figure 55*). The experiment had to be set up very carefully. (Light experiments in the lab are difficult to set up without an optical bench. Imagine how much more difficult it would be to set up on a hilltop.)

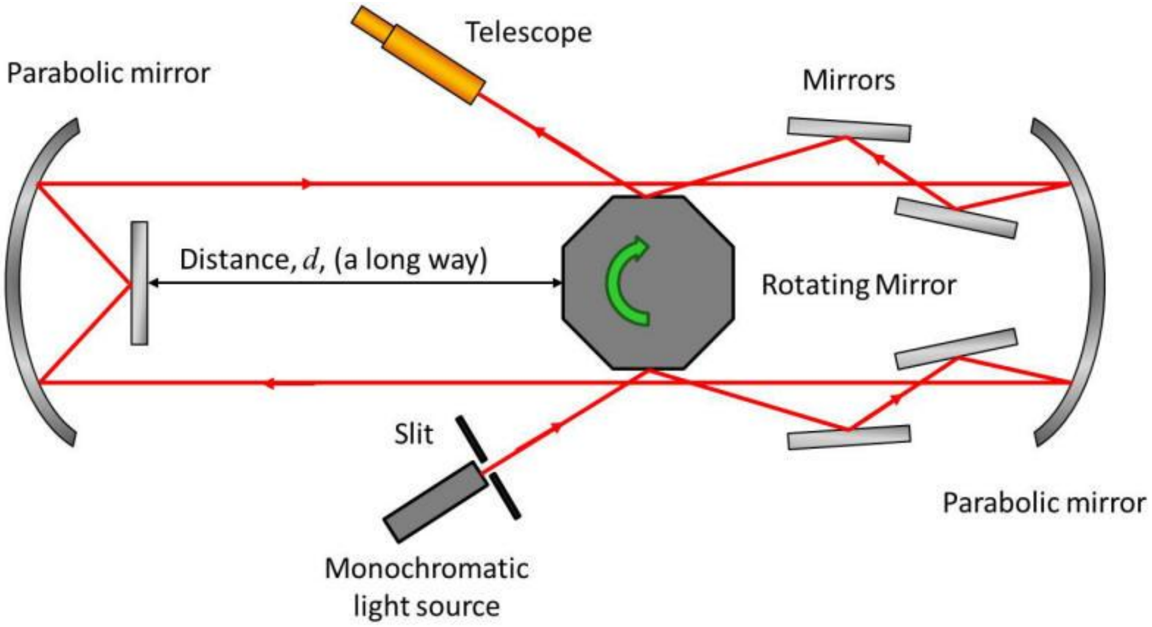


Figure 55 Michelson's experiment to measure the speed of light.

The experiment is done like this:

- The light is reflected off the bottom face of the rotating octagonal mirror.
- It passes to the parabolic mirror on the right and is reflected to the mirror on the left which is many kilometres away.
- It is reflected back to the parabolic mirror on the right and reflected to the telescope.
- When the mirror is parallel with the rays and stationary, the slit through which the light comes is visible to the observer through the telescope.
- If the mirror is slightly rotated, the beam from the source gets reflected away and does not travel to the far mirror. This is shown below (*Figure 56*):

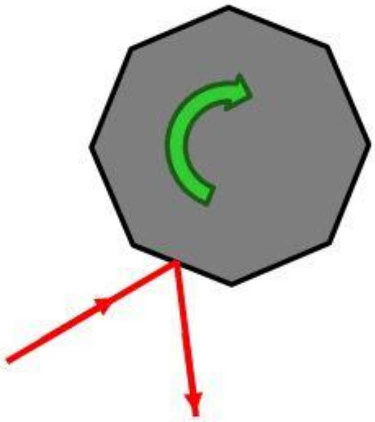


Figure 56 Light ray being reflected away from the mirror

- Therefore, the image of the slit disappears from the observer.
- If the octagonal mirror is spinning at a certain fast speed of rotation, the image appears again when the next surface appears parallel to the rays of light (*Figure 57*):

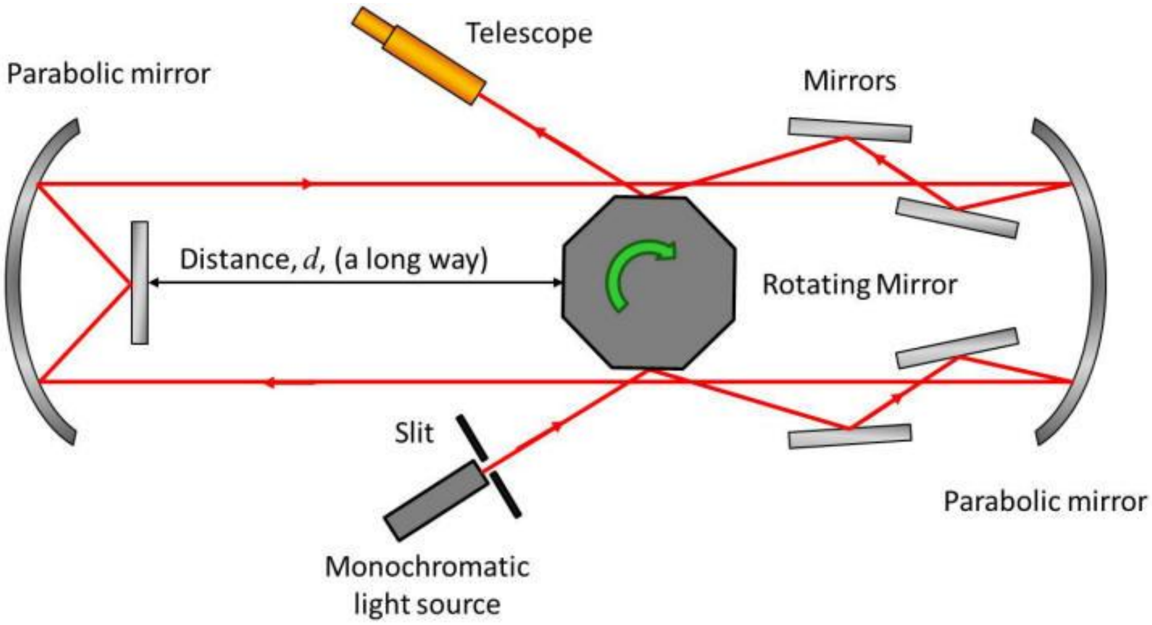


Figure 57 Light being transmitted and reflected all the way back to the observer

The appearance is quite abrupt, and the rate of turning has to be exact.

The time taken for the light to travel out and back is the same time for the mirror to rotate 1/8th of a turn. If the mirror is turning at n revolutions per second, the time taken for the light to travel is:

$$t = \frac{1}{8n}$$

.... *Equation 45*

The distance travelled by the light out and back $2d$. So, using speed = distance ÷ time, we combine *Equations 43* and *45* to give *Equation 46*:

$$c = \frac{2d}{1/8n} = 16dn$$

.... *Equation 46*

The equipment that Michelson had available was more sophisticated than Fizeau's. The octagonal mirror was spun at 512 revolutions per second using an air-turbine. The rate of turning was determined by a **stroboscope**. The distance was 36.3 km. So simple substitution gives us:

c = 16 × 36.3 × 10^3 m × 512 s^{-1} = **2.97×10^8 m s^{-1}**

Michelson's final answer was that the speed of light is **299 792 458** m s^{-1}. The quoted figure of 3.00×10^8 m s^{-1} is quite good enough for most purposes.

There were some compromises in that during the series of experiments, there was an earthquake. Some estimates suggest that the mountain may have moved by about a metre. It was not always possible to keep the mirror turning at 512 s^{-1}. Also, at that rate of turning (31 000 rpm) the mirror could fly apart, which could be highly dangerous.

In a school physics lab, it is possible to get a reasonable estimate of the speed of light in an optical fibre by sending a pulse up a length of fibre optic cable to a receiver. The transmitted and received pulses are displayed on a CRO. The time period between the pulses is measured, and the length of the fibre optic cable is measured, so it's possible to measure the speed. The diagram (*Figure 58*) shows the idea:

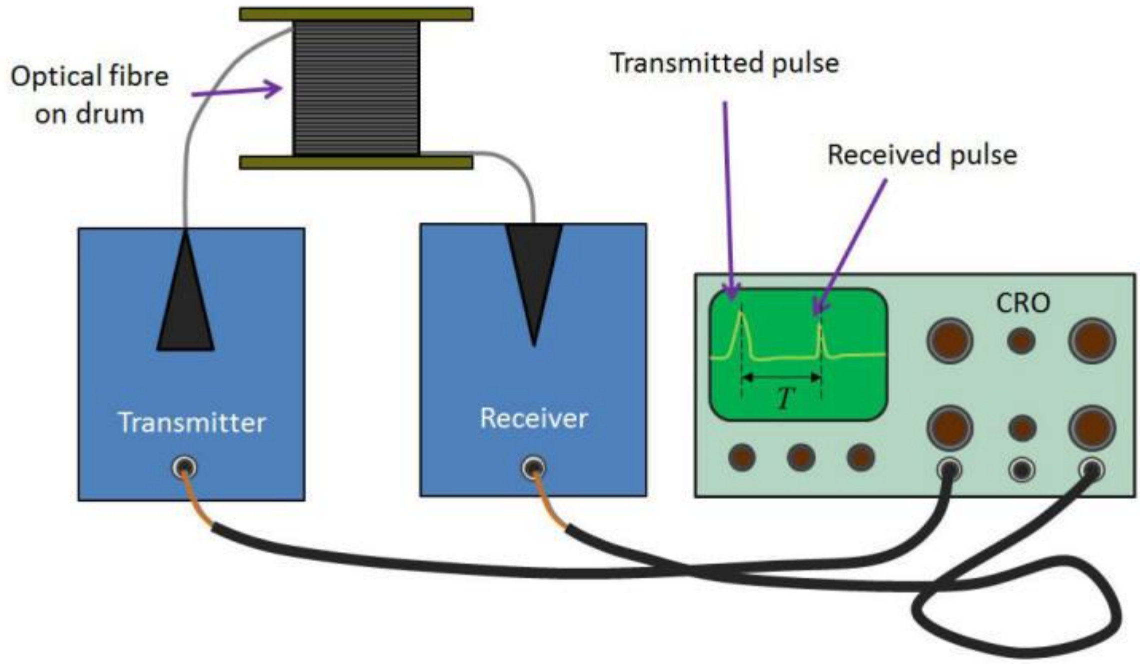


Figure 58 Measuring the speed of light in a school/college physics lab

The CRO shows the transmitted pulse, and a fraction of a microsecond later, the received pulse. The period T can be measured, and the time worked out, knowing the time-base setting.

While we may scoff at the notion of ether, we should remember that physicists had picked up their knowledge and understanding from their own teachers and colleagues. They genuinely believed it, and many found it hard to conceive that ether did not exist. We now know and take for granted the idea of light as photons that can travel in a vacuum. However, the quantum models that photons obey is difficult to understand.

We have also seen how experiments, carried out in trying conditions, using quite simple, if not primitive, equipment can give us the speed of light to a value that we now take for granted.

14D.058 Maxwell and the constancy of the Speed of Light

In Tutorial 14D.03 we saw that James Clerk-Maxwell worked out the speed of electromagnetic waves using the equation that he derived:

$$c = \frac{1}{\sqrt{(\mu_0 \varepsilon_0)}}$$

.... *Equation 47*

A simple calculation using this gives us the speed of electromagnetic waves as 2.99×10^{8} m s^{-1}. You are not expected to use the Maxwell Equations; they are difficult and will be covered at university level.

Light travels at 3.0×10^{8} m s^{-1}, regardless of whether its source is moving **towards** an observer, or **away from** the observer. Nothing can catch up with a propagating electromagnetic wave, so he argued. At those days, light was regarded as a wave. Nowadays we know that light consists of photons, which are trains of waves. **Therefore, nothing can travel at a speed greater than the speed of light**, and this formed the basis of Einstein's Theory of Special Relativity, which we will study in the next tutorial, 14D.06.

Questions

Tutorial 14D.05

14D.05.1

(a) What is the velocity of X relative to the river bed (Use a vector diagram)?

(b) What is the time for X to travel from A to B to A?

(c) What is the velocity of Y relative to the river bed going from A to C and the time?

(d) What is the velocity of the boat Y going from C to A and its time?

(e) Which boat wins and by how much?

14D.05.2

Write down Newton's First Law of Motion.

14D.05.3

Why is it not consistent with Newton I?

14D.05.4

Why is this not consistent with Newton I?

14D.05.5

What kind of frame of reference is the roundabout? Explain your answer.

14D.05.6

Why was it so hard to measure the speed of light?

14D.05.7

Look at *Figure 58*. Would this give the speed of light quoted above?

Tutorial 14D.06 Einstein's Theory of Special Relativity

AQA Syllabus

Contents

This is quite a challenging tutorial. Take it slowly and carefully.

Spoiler Alert: There is some abysmal science fiction used in some of the examples!

14D.061 Special Relativity

The Theory of Special Relativity was first published as a paper in 1905. A more general theory of relativity was brought out in 1916, which is beyond our scope. The theory of special relativity is based on two statements or **postulates**:

1. **The laws of physics have the same form in all inertial frames of reference**. This does not just apply to Newton's Laws, but all laws. No experiment could show absolute motion or absolute state of rest.
2. **The speed of light in free space is the same in all inertial frames of reference**. It does not matter whether the light is coming from a moving or stationary source, or whether the observer is moving or stationary.

The speed of light is not affected by the relativity equations we are studying in this tutorial. It shows physical **invariance**.

In special relativity, you may see the terms **proper length**, **proper time**, and **rest mass**. These are length, time, and rest mass as observed by a **stationary observer**.

Think about this (*Figure 59*):

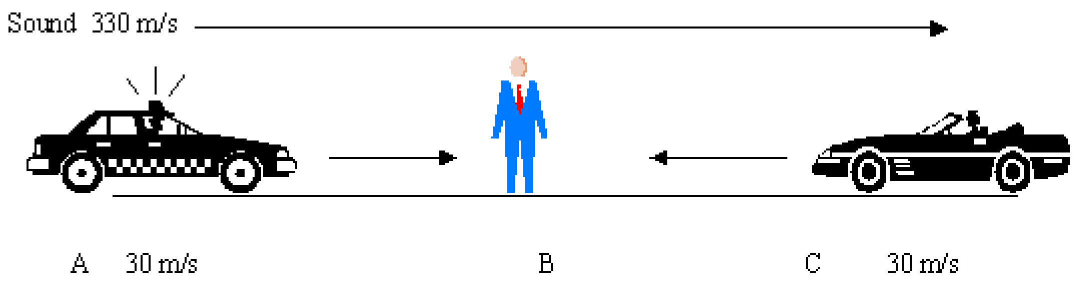


Figure 59 A simple example of the Doppler effect

The speed of sound in air is 330 m s^{-1} relative to the air.

The observer at B and at C will hear a change of frequency of the siren as the police car goes past them. This is called the **Doppler effect**.

Now consider this (*Figure 60*):

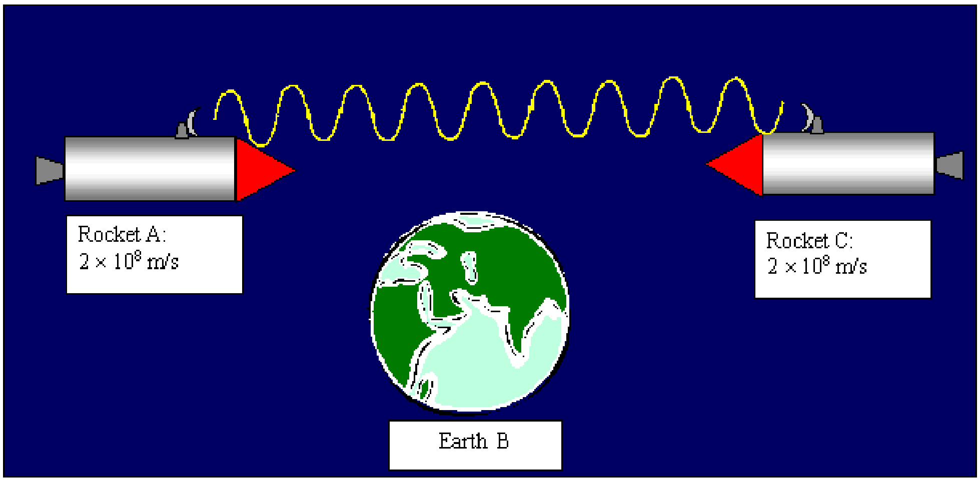


Figure 60 Two rockets approaching each other each at 2/3 speed of light.

The speed of light does not depend on the observer. All observers, whether at A, B, or C observe that light travels at 3×10^8 m s^{-1}. There is NO Doppler effect.

These postulates have far-reaching implications for Physics, and all have been confirmed by experiment. These are:

- Time dilation.
- Length contraction.

- Dependence of mass on velocity.
- Equivalence of mass and energy.
- The impossibility of acceleration beyond the speed of light.

Maths Note

The equations of relativity use the **Lorentz Transformation**. It was devised by a Dutch mathematician, Hendrik Lorentz (1853 - 1928). Initially it was little more than a mathematical curiosity - a solution waiting for a problem. It soon turned out to be the key to solving why nothing can travel faster than the speed of light. The equation is written as:

$$\gamma = \frac{1}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$

[γ - Lorentz factor; v - speed (m s^{-1}); c - speed of light (m s^{-1}).]

The term γ is "gamma", a Greek lower case letter 'g'.

It can also be written as:

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-0.5}$$

Another way of writing it is:

$$\gamma = \frac{1}{\sqrt{(1-\beta^2)}}$$

where:

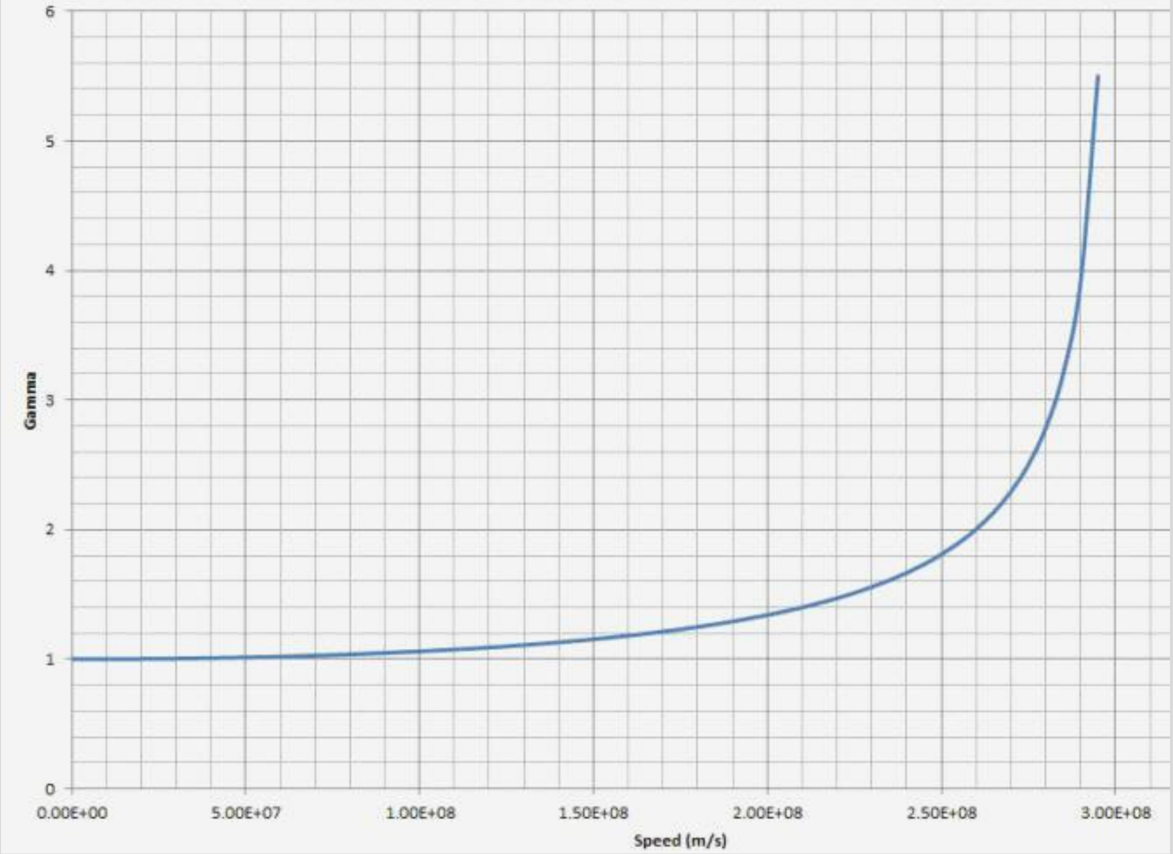
$$\beta = \frac{v}{c}$$

In some books the authors use α, the **reciprocal** of γ, to give:

$$\alpha = \frac{1}{\gamma} = \sqrt{\left(1 - \frac{v^2}{c^2}\right)}$$

This is used with length contraction. Alpha is always less than 1.

If we plot gamma against speed, the graph looks like this:



At 3.0×10^8 m s^{-1} the line would be infinity. This is not shown on this graph. From this graph we can see that the Lorentz factor is always **greater than 1.** When an object is stationary, $\gamma = 1$. When the object is travelling at less than 5.0×10^7 m s^{-1}, γ is very close to 1. The increase to anything significantly above 1 takes place at about 1×10^8 m s^{-1}.

The equation looks horrendous, but it is not difficult to work with provided you follow a **strategy**:

1. Work out the term v^2/c^2.
2. Take the number you work out away from one. You will get a **fraction**.
3. Find out the **square root** of the answer to step 2.
4. Take the reciprocal of your answer to Step 3. This is gamma (γ).
5. Now multiply whatever relativistic quantity you are working with by gamma.
6. Except for length, which you multiply by alpha.

It does not matter how you use the equation, as long as you show each step, and get the answer right!

Note that speeds of often given in terms of a fraction of c, the speed of light. Therefore, 2.40×10^{8} m s^{-1} is often given as $0.80\ c$. In the equation, the *c* terms cancel out.

14D.062 Time Dilation

If the speed of light is invariant, it can be shown that an observer will observe a moving clock as running slower than a stationary clock. This has been shown by starting two very accurate clocks at exactly the same time. This is **synchronisation** of the clocks. One of the clocks stays in the lab, while the other is taken on an aeroplane which is flown for a very long time. It is found that the moving clock is slightly behind the other clock. The difference is not big (about 1 ns). Just in case one clock was wrong, the experiment is repeated with the clocks swapped over.

Suppose the time on the stationary clock is t_0, and the time on the moving clock is t, (or t') the two are related by:

$$t = t_0\gamma$$

.... *Equation 48*

The term t_0 is the **proper time**, the time between two events as measured, as if the moving object were travelling slowly. We could call it the "rest time". The next part of the equation is called the **Lorentz factor**, and is often represented by γ, "gamma", a Greek letter 'g'. At normal speeds, $\gamma = 1$, but increases rapidly as we approach the speed of light.

This can be rewritten as:

$$t = \frac{t_0}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$

.... *Equation 49*

Or as:

$$t = t_0\left(1 - \frac{v^2}{c^2}\right)^{-0.5}$$

.... *Equation 50*

The term t is the time between the two events as measured by an observer in a frame which moves with a constant velocity v relative to the other observer.

The equation looks horrendous, but it is not difficult to work with provided you follow a strategy:

1. Work out the term v^2/c^2.
2. Take the number you work out away from one. You will get a fraction.
3. Find out the **square root** of the answer to step 2.
4. Divide the term t_0 by the answer to step 3 to get t.

The time t is always longer than t_0. If it isn't, something has gone wrong. This effect is called **time dilation**.

Worked example A spaceship passes the Earth at a speed of $0.8\ c$ ($c = 3.0 \times 10^8$ m s^{-1}) and flashes a signal lamp for 2.0 ms. What is the duration of the signal on Earth?
Answer Don't bother to convert $0.8\ c$ into m s^{-1}! It will "come out in the wash". Work out $$v^2/c^2 = (0.8\ c)^2 \div (1\ c)^2 = 0.64$$ Take away 0.64 from 1: $$1 - 0.64 = 0.36$$ Take the square root of 0.36: $$(0.36)^{0.5} = 0.60$$ Now work out γ: $$\gamma = 0.60^{-1} = 1.667$$ Since: $$t = t_0\gamma$$

Time

$$t = 2.0 \text{ ms} \times 1.667 = \underline{\mathbf{3.33\ ms}}$$

(Author's note: I would like to thank my student Joseph Reffitt who got me thinking hard about the wording of this example.)

Not that hard, is it?

Time dilation has been shown even in relatively slow-moving objects like aeroplanes. A clock was taken up in an aeroplane and flown about for several hours, while a second identical clock was left running on the ground. There was a tiny but measurable difference between the two. In our sedate lifestyles, the difference is so tiny as to be negligible.

Muon decay gives us more tangible evidence for time dilation. Muons (see Particle Physics) are unstable subatomic particles that are formed in the upper atmosphere by cosmic rays.

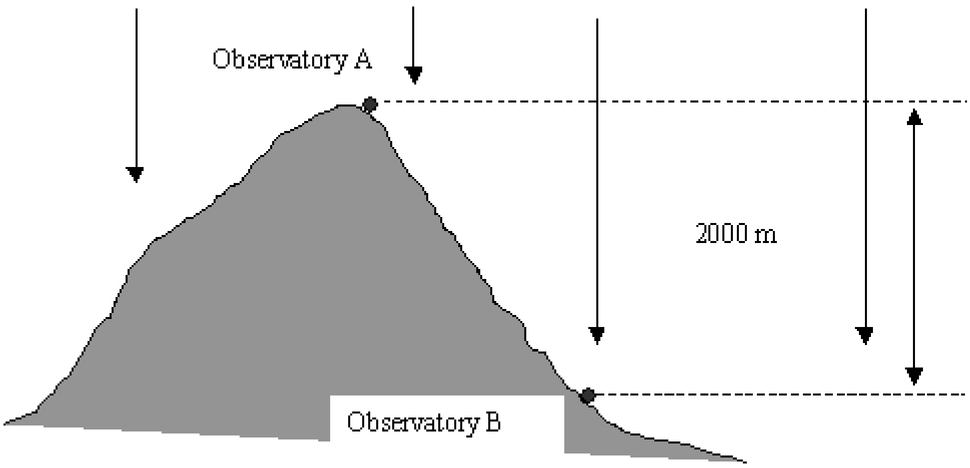


Figure 61 Muons detected at the top and bottom of a mountain

Muons can be detected at an observatory at the top of a mountain, at A, and at the bottom of the mountain at B. Here are some data about muons:

- Intensity of muons at B = 80 % intensity at A.
- Half-life of muons at rest = 2.20 ms.
- Speed of muons = $0.996\ c$.

14D.063 Length Contraction

Another consequence of the invariance of the speed of light is that an observer measuring a rod moving parallel to its length will find that it is shorter relative to its stationary length. In effect if you measure a moving car, you will find that it is shorter than the stationary car. This effect is called **length contraction**.

As with time dilation, the change is so tiny as to be negligible. But this is not the case for objects moving close to the speed of light.

The relationship is a little easier than the one for time:

$$l = \alpha l_0$$

.... *Equation 51*

(See the Maths Note above for what is meant by alpha)

This can be written as:

$$l = l_0 \left(1 - \frac{v^2}{c^2}\right)^{0.5}$$

.... *Equation 52*

This is more easily written as:

$$l = l_0 \sqrt{\left(1 - \frac{v^2}{c^2}\right)}$$

.... *Equation 53*

The term l_0 is the proper length as measured in the frame that is at rest relative to the object.

The term l is the length as measured by an observer in a frame of reference that moves at a constant relative velocity of v.

Again, this looks like a fairly horrendous equation, but use the problem-solving strategy below and it's not that difficult:

1. Work out the term v^2/c^2.

2. Take the number you work out away from one. You will get a fraction.

3. Find out the **square root** of the answer to step 2.

4. Multiply the term l_0 by the answer to step 3 to get l.

Your moving length will be less than the stationary length. If it's more, you've made a mistake.

Let's go back to our mountain (*Figure 62*):

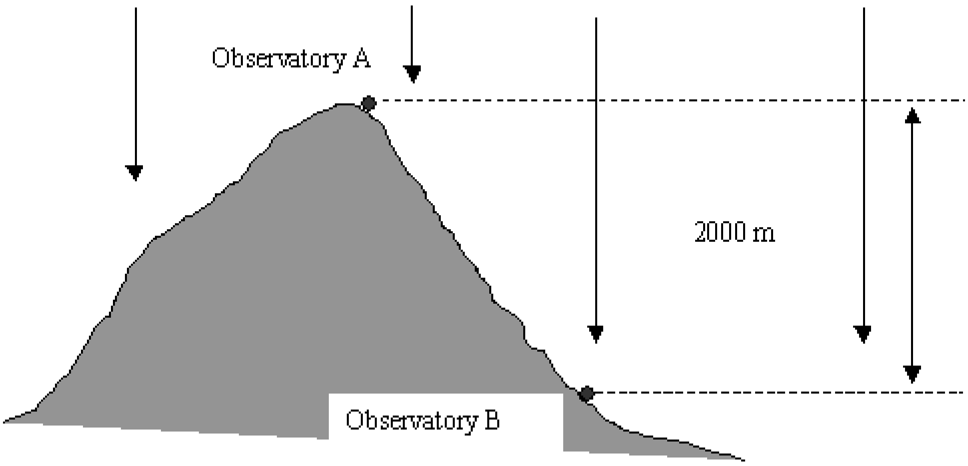


Figure 62 Muons make a mountains seem less high

Muon decay can be explained by applying the **contraction equation**. We consider the problem from the **moving frame of reference** of the muon instead of the fixed frame of reference of the Earth. In this case the muon sees the Earth and the mountain as a giant pancake with a pimple on it. Let's remind ourselves of the data:

- Intensity of muons at B = 80 % intensity at A.
- Half-life of muons at rest = 2.20 ms.
- Speed of muons = $0.996\ c$.

Does this mean that my 30 km train journey to work in the morning is shorter than it really is? In theory yes, by less than 1 micrometre.

14D.064 Mass Increase

Two key points

- The speed of light is invariant.
- Momentum is conserved.

Therefore, a stationary observer will measure the mass of a moving object as being greater than that object when it is stationary relative to the observer. This is described in the equation:

$$m = \gamma m_0$$

.... *Equation 54*

Or:

$$m = \frac{m_0}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$

.... *Equation 55*

The term m_0 is called the proper or **rest mass**. It is the mass as measured by an observer in a frame of reference which is at rest relative to the observer.

The term m is the relativistic mass which is the mass as measured by an observer in a frame that is moving at a constant velocity v.

The equation looks horrendous, but you should be getting used to these equations by now. It is not difficult to work with provided you follow a strategy:

1. Work out the term v^2/c^2.
2. Take the number you work out away from one. You will get a fraction.
3. Find out the **square root** of the answer to step 2.
4. Take the reciprocal to get γ.
5. Multiply the term m_0 by γ to step 4 to get m.

The time m is always bigger than m_0. If it isn't, something has gone wrong.

14D.065 Impossibility of Speeds Greater than the Speed of Light

As v approaches c, the mass m gets bigger and bigger. The closer it gets, the more it tends to infinity. Further acceleration requires a force approaching infinity. So, it is impossible for the speed of light to be reached, let alone exceeded by an object of non-zero rest mass. We can even plot a graph (*Figure 63*):

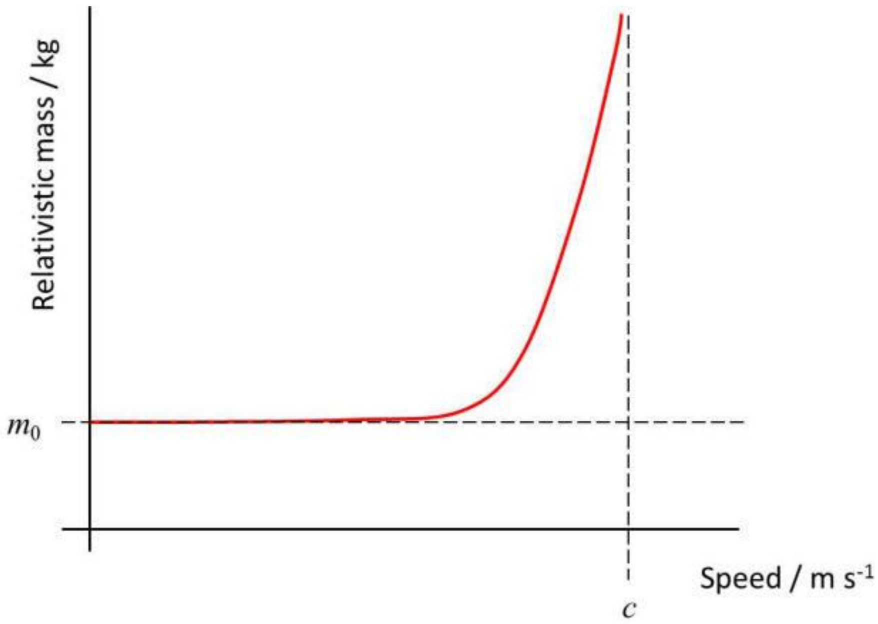


Figure 63 Increase in mass as an object approaches the speed of light

Now think about this (*Figure 64*):

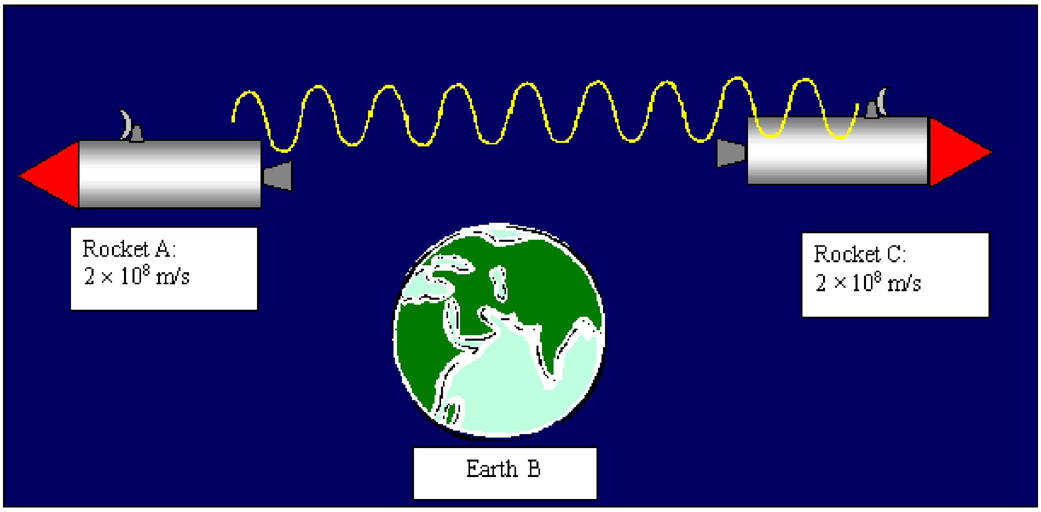


Figure 64 Two rockets fly away from each other, each travelling at 2/3 c

Although the immediate and natural reaction is to say that the speed of C relative to A is 4×10^8 m s^{-1}, or 4*c*/3, it can be shown using equations similar to the ones we have seen above that the relative velocities of the two rockets are $12c/13$. (See 14D.610 at the end of this tutorial.) Therefore, the radio waves from A can catch up with C. This requires more complex relationships.

This is above the speed of light, but no mass is being transferred so it is possible.

Particles with zero rest mass, photons and neutrinos, always travel at the speed of light. It has been suggested that there may be a group of particles called **tachyons** for which $v > c$ at the instant they are created. They always travel faster than light and speed up as they lose energy. They have never been detected.

14D.066 Mass and Energy

In classical physics, we know that kinetic energy is given by:

$$E_k = \frac{1}{2}mv^2$$

.... *Equation 56*

If we accelerate an electron with a voltage V, we know that all the energy is kinetic, so we can write:

$$eV = \frac{1}{2}mv^2$$

.... *Equation 57*

If we plot a graph of speed against accelerating voltage, we see (*Figure 65*):

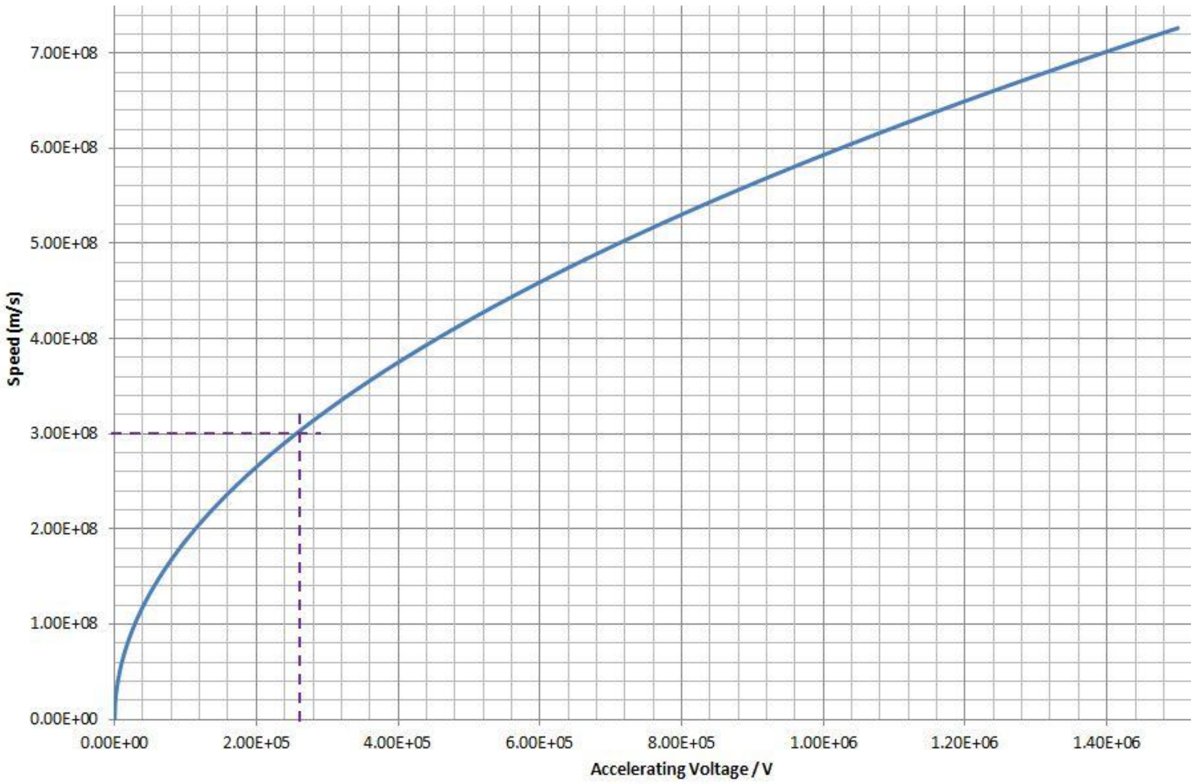


Figure 65 Graph of electron speed against accelerating voltage (Classical physics)

We see from this graph that at about 2.6 × 10^5 V, the electron speed is 3.0 × 10^8 m s^{-1}.

Since nothing can travel at more than the speed of light, clearly the classical model has broken down, so a more sophisticated approach is called for. Einstein's Theory of Relativity provided the answer in that if energy is supplied to an object, its mass increases. Conversely, if energy is transferred away from the object, the mass decreases. Therefore, as an object speeds up, its kinetic energy increases and so does its mass. According to the theory of special relativity, the increase in energy is proportional to the increase in mass:

$$\Delta E \propto \Delta m$$

$$\Delta E = \Delta mc^2$$ .... *Equation 58*

In this case c^2 (= 9.00×10^{16} m^2 s^{-2}) is the constant of proportionality. This can be extended to give us the total energy of a particle:

- A particle at rest has **rest energy**

$$E_0 = m_0c^2$$ .... *Equation 59*

- The moving electron has additional energy as kinetic energy only, which is given by:

$$E_k = eV$$ .... *Equation 60*

- Total energy = Rest energy + kinetic energy. [Learn this; it is useful].

$$E = E_0 + E_k$$ .... *Equation 61*

$$E = m_0c^2 + eV$$ .... *Equation 62*

- If the same particle is moving at constant velocity v, the relativistic mass, m is given by:

$$m = \frac{m_0}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$

.... *Equation 63*

- So, we can write an expression for the **total energy** using:

$$E = mc^2 = \gamma m_0 c^2$$

.... *Equation 64*

Therefore:

$$E = \frac{m_0 c^2}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$

.... *Equation 65*

And we can write an equation for the total energy in terms of the rest energy and the kinetic energy:

$$(m_0 c^2 + eV) = \frac{m_0 c^2}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$

.... *Equation 66*

This again looks horrendous, but apply the problem-solving strategy:

1. Work out the term v^2/c^2.

2. Take the number you work out away from one. You will get a fraction.

3. Find out the **square root** of the answer to step 2.

4. Divide the term m_0 by the answer to step 3 to get m.

Worked example

Calculate the speed of an electron which has been accelerated from rest through a p.d of 2.0×10^6 V

What NOT to do!

$$eV = \tfrac{1}{2} mv^2$$

$$v^2 = (2 \times 1.6 \times 10^{-19}\ \text{C} \times 2.0 \times 10^6\ \text{V}) \div 9.11 \times 10^{-31}\ \text{kg}$$

$$= 7.02 \times 10^{17}\ \text{m}^2\ \text{s}^{-2}$$

$$v = 8.38 \times 10^8\ \text{m s}^{-1}$$

Answer

Total energy = rest energy + kinetic energy

Kinetic energy = charge × voltage

Kinetic energy = $eV = 1.6 \times 10^{-19}\ \text{C} \times 2.0 \times 10^6\ \text{V} = 3.2 \times 10^{-13}\ \text{J}$

Rest energy = $m_0c^2 = 9.11 \times 10^{-31}\ \text{kg} \times 9.0 \times 10^{16}\ \text{m}^2\ \text{s}^{-2} = 8.2 \times 10^{-14}\ \text{J}$

Total energy = $3.2 \times 10^{-13}\ \text{J} + 8.2 \times 10^{-14}\ \text{J} = 4.02 \times 10^{-13}\ \text{J}$

Now use:

$$E = \frac{m_0c^2}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$

Substituting gives us:

$$4.02 \times 10^{-13}\text{J} = \frac{8.20 \times 10^{-14}\text{J}}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$

Rearranging gives us:

$$\sqrt{\left(1 - \frac{v^2}{c^2}\right)} = \frac{8.20 \times 10^{-14}\text{J}}{4.02 \times 10^{-13}\text{J}} = 0.204$$

Square this to get rid of square root:

$$\left(1 - \frac{v^2}{c^2}\right) = 0.0416$$

Now rearrange

$$-\frac{v^2}{c^2} = 0.0416 - 1 = -0.9584$$

$$\frac{v^2}{c^2} = 0.9584$$

$$v^2 = 0.9584\, c^2$$

The square root gives us:

$v = 0.979\, c =$ **2.9×10^8 m s^{-1}**

There are a lot of steps in a calculation like this, but they are quite simple, so don't panic when you see an example like this.

Now you have a go at Question 14D.06.10.

14D.067 Bertozzi's Experiment

In 1964, an American physicist, William Bertozzi, carried out an experiment to determine the relationship with speed and kinetic energy of accelerated electrons. He used apparatus like this simplified arrangement (*Figure 66*):

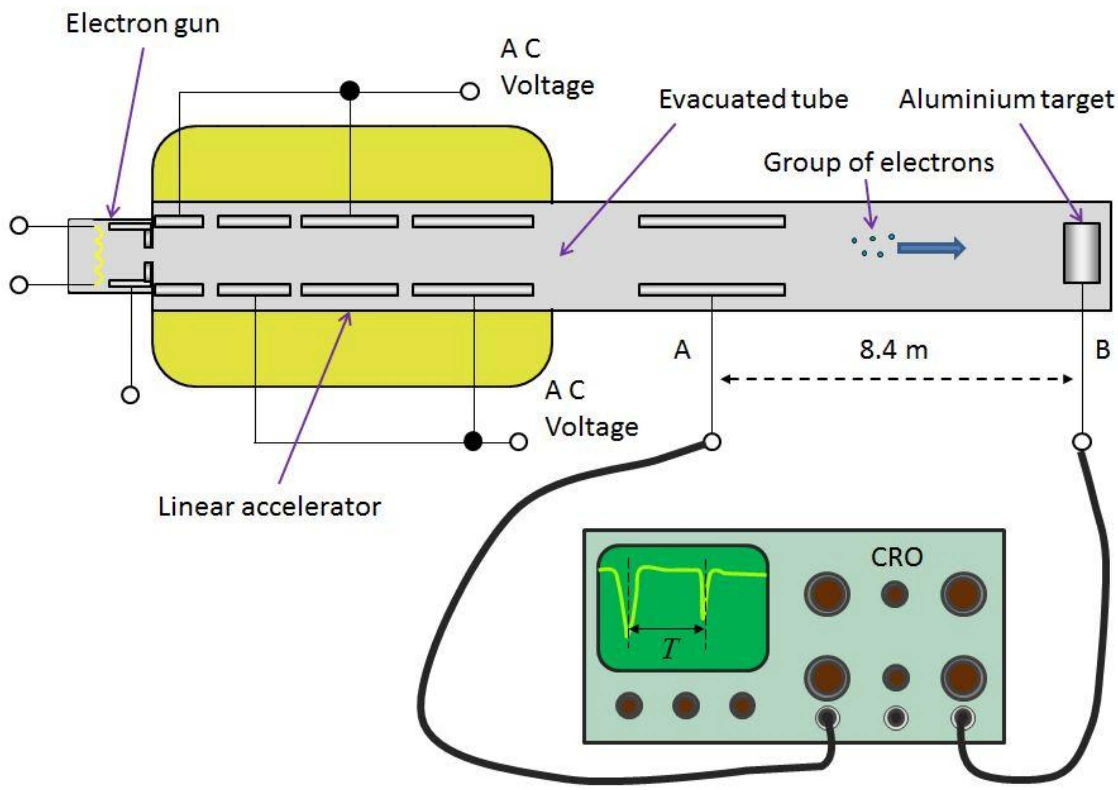


Figure 66 Bertozzi's experiment

Electrons from an electron gun are fired in bursts and they travel as a group in an evacuated tube. They are accelerated using a linear accelerator. They pass through a pair of plates at A and are detected as a pulse on the CRO. They then strike an aluminium target at B, and a second pulse is shown on the CRO. The main point of this experiment is that the actual kinetic energy of the electrons is measured by **measuring** the temperature rise of the aluminium target at B. Previously the kinetic energy had been inferred from:

$$E_k = eV$$ .... *Equation 67*

Instead, the electrons give up their kinetic energy to the aluminium target of mass m, and specific heat capacity c. We then work out the energy transferred to the target by measuring its temperature change. If we know how many electrons there are in each group, we can work out the kinetic energy for each electron. The energy going into the target is:

$$\Delta E = mc\Delta\theta$$

.... *Equation 68*

If there are n electrons, we can say that the kinetic energy for each electron is:

$$E_\mathrm{k} = eV = \frac{mc\Delta\theta}{n}$$

.... *Equation 69*

Do not confuse the code c used here for specific heat with c used for the speed of light.

We can generate a graph of kinetic energy against speed:

Kinetic energy = Total energy - rest energy

In physics code:

$$E_\mathrm{k} = \frac{m_0c^2}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}} - m_0c^2$$

.... *Equation 70*

The graph shows the kinetic energy in eV against the speed of the electron (*Figure 67*):

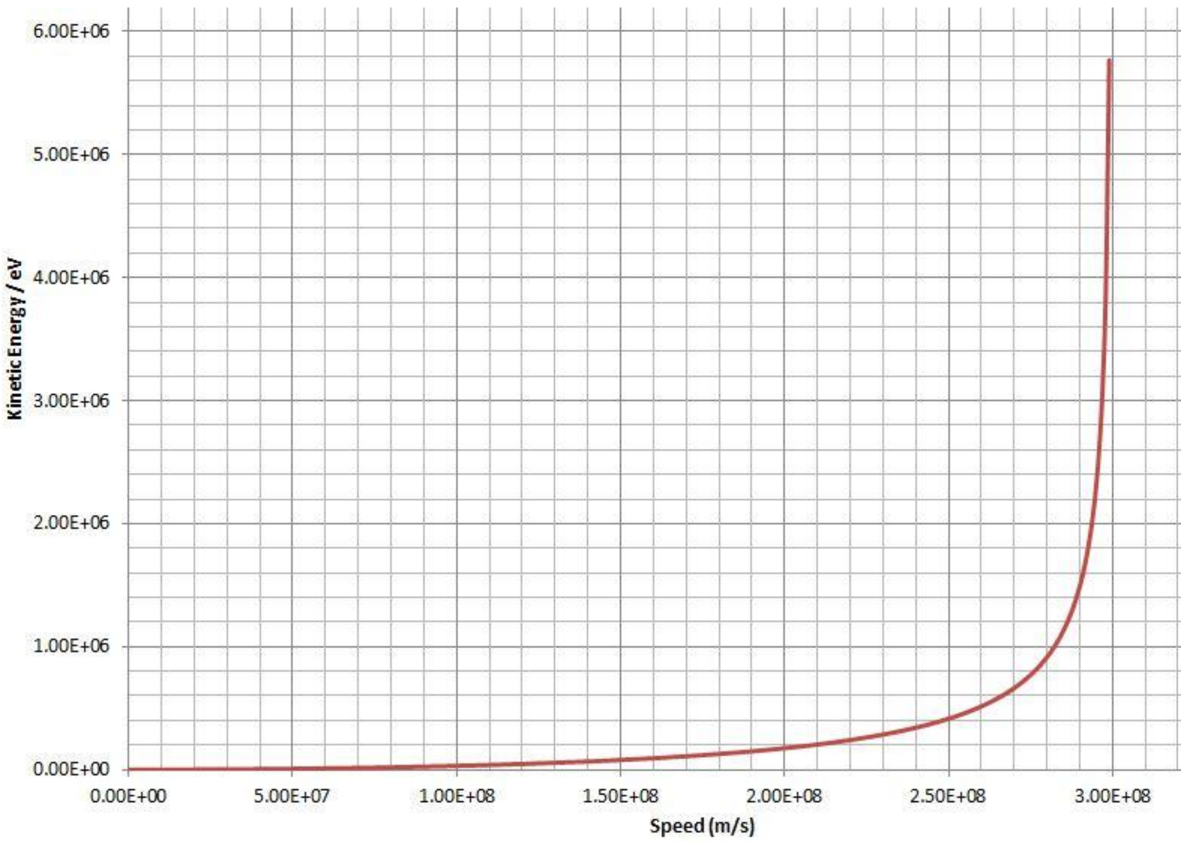


Figure 67 Graph of electron kinetic energy against electron speed

Bertozzi's results were within 10 % of the idealised data that were used to generate the graph above. The conclusion was that as the kinetic energy of the electrons increased, the speed of the electrons approached a limiting speed. Therefore, nothing can exceed the speed of light.

Note that the traditional formula for kinetic energy:

$$E_k = \frac{1}{2}mv^2$$

.... *Equation 71*

is a solution for:

$$E_k = \frac{m_0c^2}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}} - m_0c^2$$

.... *Equation 72*

when the speed is very much less than the speed of light. You are NOT expected to know this for the exam.

14D.068 A Fly in the Ointment?

In Summer 2011 observations suggested that neutrinos could travel faster than light. The neutrinos had their source at the CERN experiment and travelled to be detected at a laboratory in Italy, 730 km away. The little brutes were seen to arrive 60 nanoseconds before the expected time. Many physicists remained skeptical, but the results were repeated. If it were to be concluded that neutrinos could travel faster than light, then a fundamental tenet of physics would have been undermined.

Later it was discovered that there was a faulty connection between a GPS device and a computer, so the data were wrong all the time. Neutrinos don't travel faster than the speed of light. That's the little brutes put back in their place. At least one physicist ended up "on the carpet".

14D.069 Forces on a Charge from an Electric Current (*IB students*)

We know that when there is an electric current flowing through a wire, there is always a magnetic field. It is a consequence of **special relativity**. Remember that time and length are not absolute. They are observed differently depending on whether the observer is in a stationary or moving frame of reference:

- **Time** passes more slowly for an observer in a moving frame of reference than for an observer in a stationary frame of reference. This is **time dilation**.
- **Length** contracts in a moving object. When a stationary observer views a moving object, it is shorter than the length of the object for a moving observer. This is length contraction.

The argument goes like this. Consider this piece of copper wire (*Figure 68*). It is long and straight. It is **neutral** as there are equal numbers of positive and negative charges. (The model is a lattice of ions in a sea of free electrons.) We can show them arranged like this for simplicity:

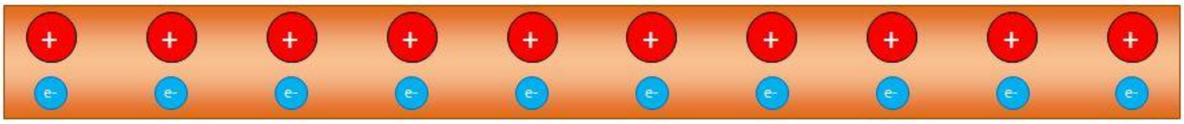


Figure 68 A neutral copper wire (simplified)

If we bring a positive charge to the wire, there is no electrical force (*Figure 69*):

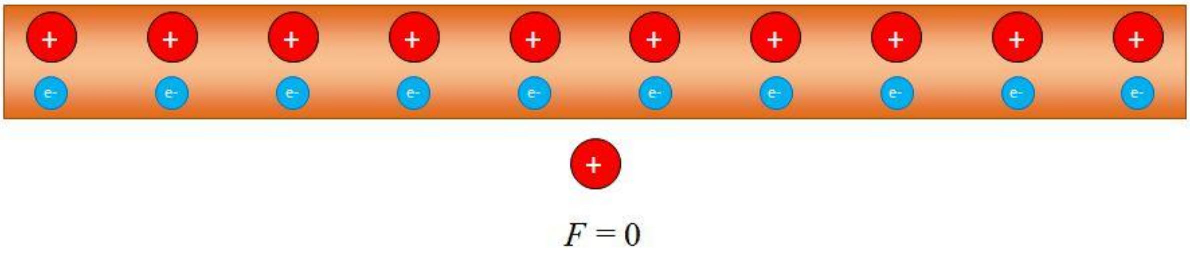


Figure 69 Bringing a positive charge to the wire. There is no electrical force.

Now suppose we connect the wire to a battery. The electrons move to the right as shown (*Figure 70*).

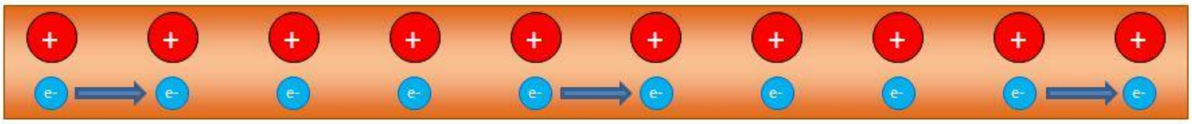


Figure 70 Current in the wire results in a flow of electrons

If we bring a positive charge to the wire, there is no **electrical** force (*Figure 71*):

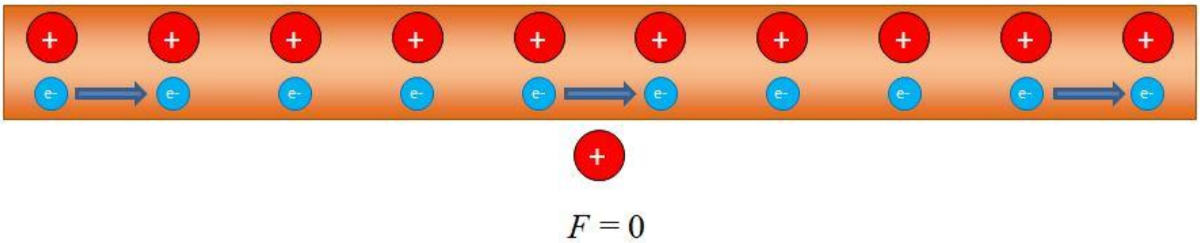


Figure 71 There is still no force on a positive charge

Although the electrons are moving, the charge density remains the same, so there is no **electrical** force acting on the charge.

Now suppose we move the electric charge at the same velocity as the electron flow (*Figure 72*):

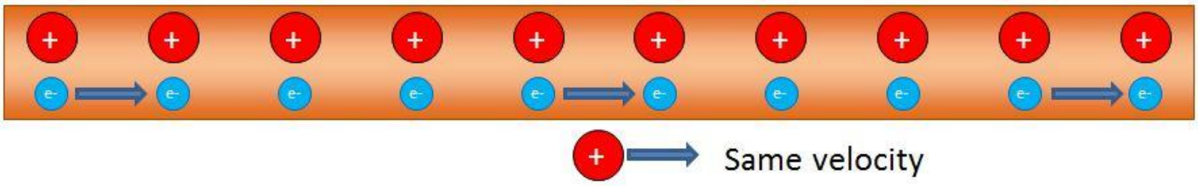


Figure 72 Moving the positive charge at the same velocity as the electrons

For the stationary observer, there is no change in the charge density, and the wire is still neutral, so there is no electrical force.

Now let's move to the **frame of reference of the test charge**. We are observing from the point of view of the charge (*Figure 73*).

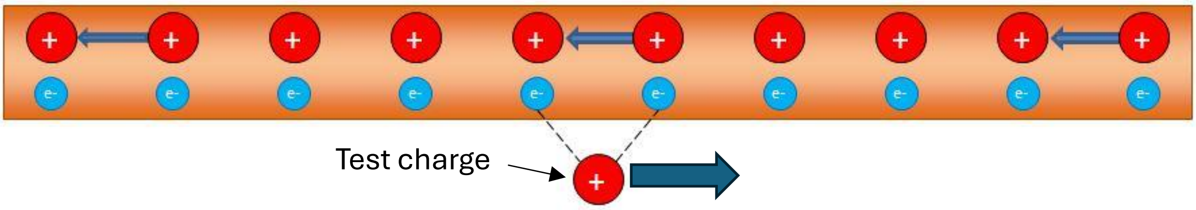


Figure 73 Viewing the situation from the frame of reference of the moving positive charge

As far as the charge sees it, the positive charges are moving from right to left. Since they are moving, the distance between the positive charges is reduced as a result of their motion. The motion is not at all fast, but the principle of length contraction still applies. Also, since the electrons are stationary, they are spread out. So, the situation according to the charge is this (*Figure 74*):

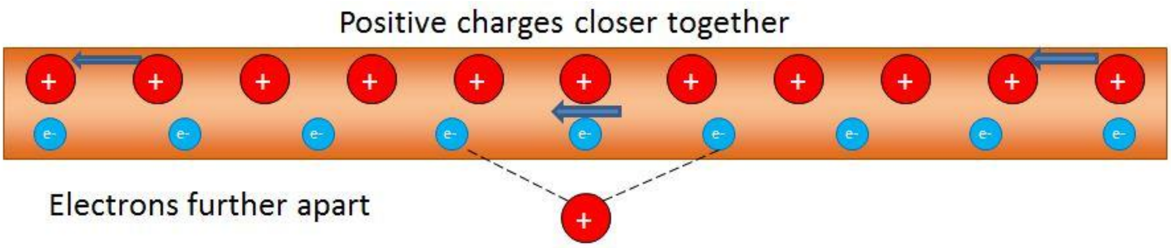


Figure 74 The positive charges appear closer together to the moving test charge

Therefore, the positive charges have a **higher charge density** than the than the electrons, so to the charge the wire is not neutral, and gives out a **repulsive** electrical force (*Figure 75*):

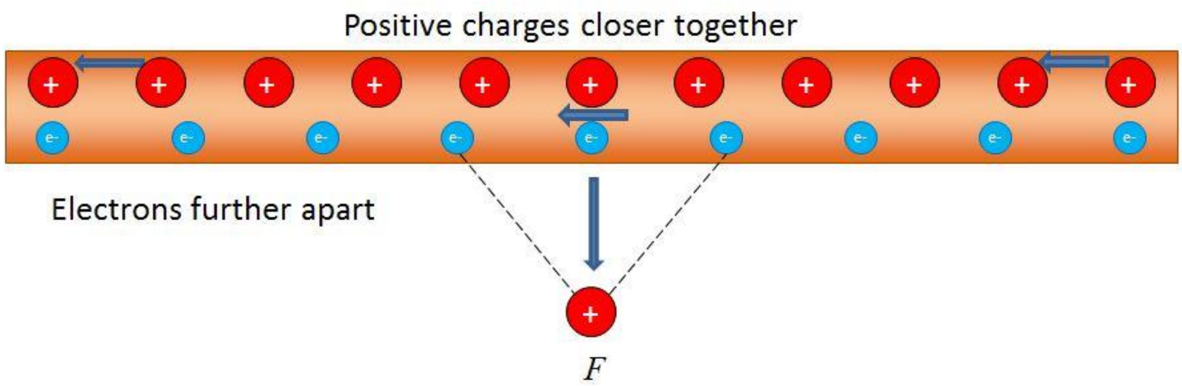


Figure 75 Thus the test charge experiences a force.

Now the stationary observer will see the effect of the force on the charge. And being a good classical physicist, the observer will conclude that the force is moving due to the action of a **magnetic** field.

The electron drift speed in copper is less than 1 mm s^{-1}. The length contraction would be very small, and normally we would ignore it. However small it is, it is still definite. But we see a measurable effect because:

- There are lots of electrons.
- The electric interaction is very strong.

From special relativity, we can conclude that **magnetic** fields observed from a **stationary** frame of reference are actually **electric** fields from a **moving** frame of reference.

Not exactly intuitive, is it?

14D.0610 Einstein Velocity Additions (*IB students*)

As we saw above, the maximum relative velocity is c, 3.0 × 10^8 m s^{-1}. If an object is travelling at $0.6\ c$, and another object is approaching at $0.8\ c$, the relative approach speed is NOT $1.4\ c$, but $1.0\ c$. We can obtain expressions for relative velocities for moving objects. These are called **Einstein Velocity Additions**.

Consider this situation in *Figure 76*. (*Spoiler alert - some very bad science fiction*):

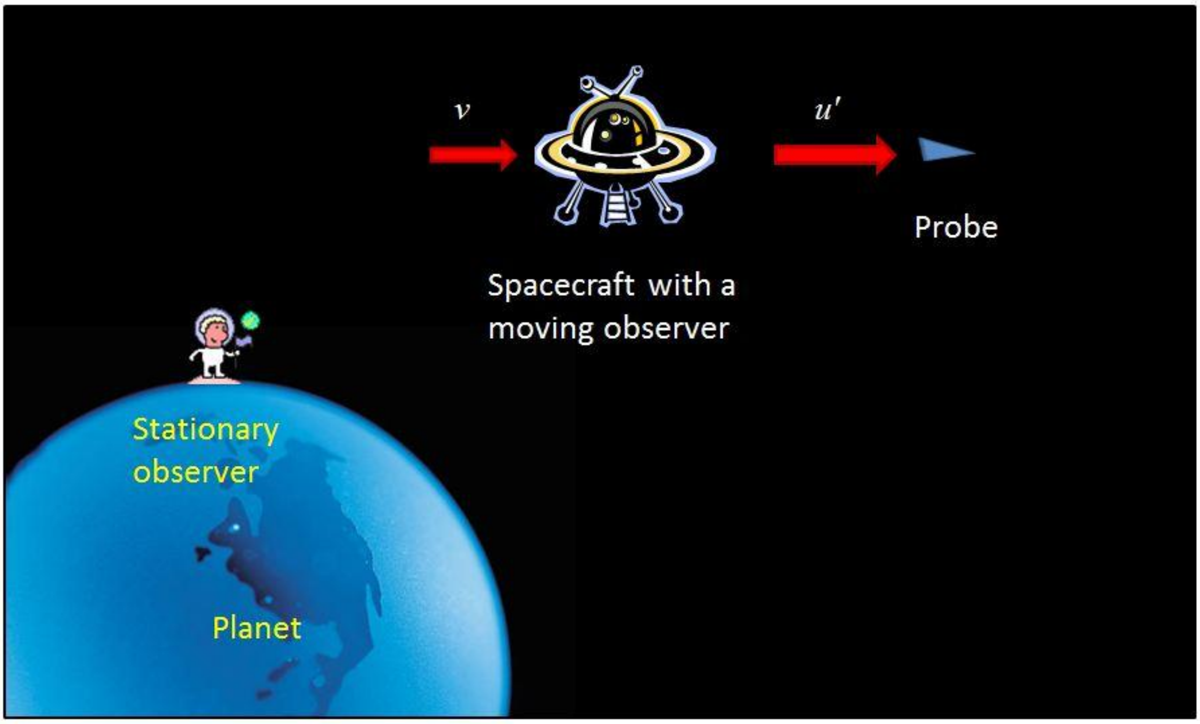


Figure 76 Thinking about Einstein Velocity Additions

A **stationary observer** is on a far-distant planet. She sees a spacecraft travelling at a velocity of v. It fires a probe in the direction it is travelling. To the stationary observer the velocity is u. In the spacecraft there is a **moving observer**. He sees that the probe has a velocity of u'.

To the **stationary observer** the velocity of the probe u can be worked out as:

$$u = \frac{v + u'}{1 + \frac{vu'}{c^2}}$$

.... *Equation 73*

To the **moving observer**, the velocity of the probe is u', which is worked out as:

$$u' = \frac{u - v}{1 - \frac{uv}{c^2}}$$

.... *Equation 74*

Worked example The spacecraft in the example above has a velocity $0.40\ c$ as measured by the stationary observer on the planet. The probe has a velocity of $0.50\ c$ as measured by the stationary observer. What is the velocity of the probe as measured by the moving observer in the spacecraft?
Answer Work out the term uv/c^2 = (0.40 *c* × 0.50 *c*) ÷ c^2 = 0.20 The value downstairs = 1 - 0.20 = 0.80 Therefore: u' = (0.50 *c* - 0.40 *c*) ÷ 0.80 = 0.125 *c*

Spacecraft Moving in Opposite Directions

In the last example, the directions were all positive, i.e. from left to right. Now let's consider two spacecraft approaching each other in opposite direction, as in the picture (*Figure 77*).

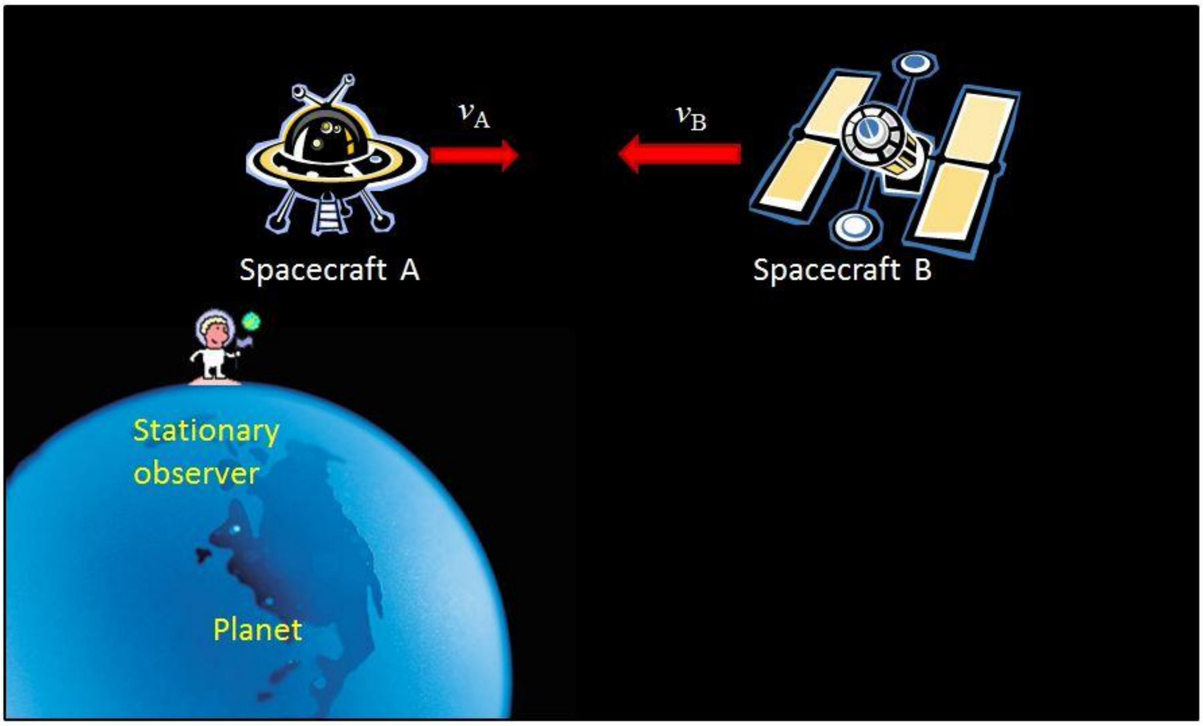


Figure 77 Two spacecraft moving in opposite directions

Our initial reaction is to say that the approach speed is given by:

$$v_{app} = |v_A| + |v_B|$$

............*Equation 75*

But we can't use that in a relativity context. The straight brackets show that we are talking about the values of the velocities (i.e., speeds).

Suppose we have an observer on spacecraft A. The relativity equation is as before:

$$u' = \frac{u - v}{1 - \frac{uv}{c^2}}$$

.... *Equation 76*

The u' is because we are considering the moving frame of reference.

The stationary observer will see the velocity of spacecraft B as v_B, therefore $u = v_B$. She will also see the velocity of spacecraft A as v_A, therefore $v = v_A$.

The moving observer in spacecraft A will see the velocity of spacecraft B as u'.

So, we can change the equation to:

$$u' = \frac{v_B - v_A}{1 - \frac{v_B v_A}{c^2}}$$

.... *Equation 77*

Let's put some numbers in to see what happens:

Worked Example The spacecraft in the example above has a velocity 0.60 *c* as measured by the stationary observer on the planet. The probe has a velocity of 0.50 *c* as measured by the stationary observer. What is the velocity of the probe as measured by the moving observer in the spacecraft?
Answer Use: $$u' = \frac{v_B - v_A}{1 - \frac{v_B v_A}{c^2}}$$ Work out the term uv/c^2 = (0.60 c × -0.50 c) ÷ c^2 = -0.30 The value downstairs = 1 - -0.30 = 1.30 Therefore: u' = (0.60 c - -0.50 c) ÷ 1.30 = 1.10 c ÷ 1.30 = **0.85 c**

If the velocities of the spacecraft are very much less than the speed of light, the "common sense" relationship is true.

$$v_{\mathrm{app}} = |v_{\mathrm{A}}| + |v_{\mathrm{B}}|$$

.... *Equation 78*

Further material on Special Relativity can be found in Topic 15, Tutorials 1 and 18.

Questions

Tutorial 14D.06

14D.06.1

Look at *Figure* 59. What is the speed of sound relative to:

A:

B:

C?

14D.06.2

What is a muon?

14D.06.3

Use the data on Page 89 above to work out:

(a) The time taken for a muon to travel 2000 m.

(b) The number of rest half-lives elapsed in this time.

(c) The expected intensity at B compared with that at A.

(d) The half-life of moving muons relative to a stationary observer.

(e) The number of moving half-lives elapsed in the time worked out in (a)

(f) The intensity at B assuming time dilation.

14D.06.4

A spaceship of length 60.0 m passes the Earth at a speed of $0.98\ c$. What is the length of the spaceship as seen by an observer on Earth?

14D.06.5

An observer measures the length of a metre rule to be 80 cm. What is the speed relative to the observer? ($c = 3.0 \times 10^{8}$ m s^{-1})

14D.06.6

Use the data on Page 91. Calculate:

(a) the height of the mountain as it passes a muon.

(b) the time taken for the mountain to pass the muon.

(c) the number of rest half-lives elapsed in the time in part (b)

(d) the percentage of muons remaining when point B is passed compared to point A

14D.06.7

The rest mass of an electron is 9.11×10^{-31} kg at rest. What is its mass when it is travelling at $0.998\ c$?

14D.06.8

A powerful laser is set up on Earth which can shine a spot of light on the moon. Suppose this laser is set up so that it sweeps through 180° a second, i.e. 30 rpm. What is the speed at which the beam sweeps across the surface?

Distance from Earth to the Moon 4×10^{8} m.

14D.06.9

Show that, when an electron is travelling at 3.00×10^{8} m s^{-1}, the accelerating voltage is about 2.6×10^{5} V.

Mass of electron = 9.11×10^{-31} kg.

Magnitude of the charge of an electron = 1.60×10^{-19} C.

14D.06.10

What p.d. is needed to accelerate a proton, rest mass 1.67×10^{-27} kg from rest to a speed of 0.95 c?

14D.06.11

1.2×10^{15} electrons are accelerated by an accelerating voltage of 2.50×10^{6} V. They strike an aluminium plate of mass 100 g.

(a) Calculate the charge in coulomb (C)

(b) Calculate the temperature change.

Magnitude of electronic charge = 1.60×10^{-19} C.

Specific heat capacity of aluminium = 900 J kg^{-1} K^{-1}.

14D.06.12

Use the value of u' in the worked example on Page 108 to verify that the stationary observer's measurement of the velocity of the probe is correct at $0.5\ c$.

Answers to Questions

Tutorial 14D.01

14D.01.1

Cathode:

Source of electrons
which are boiled off by the heating element (thermionic emission

Anode:

Attracts the electrons
because it is positively charged

14D.01.2

Gas molecules would cause collisions
and reduce the energy of the electrons.

14D.01.3

(a)

$E_k = eV$ = 1.60 × 10^{-19} J eV^{-1} × 400 V = 6.40 × 10^{-17} J

$v^2 = 2E_k/m$ = 2 × 6.40 × 10^{-17} J ÷ 9.11 × 10^{-31} kg = 1.41 × 10^{14} m^2 s^{-2}

v = **1.18 × 10^7 m s^{-1}**

(b)

$E_k = eV$ = 1.60 × 10^{-19} J eV^{-1} × 400 000 V = 6.40 × 10^{-14} J

$v^2 = 2E_k/m$ = 2 × 6.40 × 10^{-14} J ÷ 9.11 × 10^{-31} kg = 1.41 × 10^{17} m^2 s^{-2}

v = 3.74 × 10^8 m s^{-1}

14D.01.4

Magnetic Field: Path is circular
Electric Field: Path is parabolic.

14D.01.5

Resultant force is Zero
The electron carries on in a straight line.

You tell this by there being a spot on the phosphor screen.

14D.01.6

Easy to measure directly: separation of the plates

Hard to measure directly: velocity of the electron
B field (It needs an equation to measure)

Of constant value: mass of the electron
Charge of the electron

14D.01.7

Specific charge = e/m = -1.6×10^{-19} C ÷ 9.11×10^{-31} kg

= **-1.76×10^{11} C kg^{-1}**

(Note the minus sign as the electron is negatively charged)

Value is constant.

The value would be the same for a positron,

since charge and mass are same.

But the sign would be positive.

14D.01.8

Specific charge = $+1.60 \times 10^{-19}$ C ÷ 1.67×10^{-27} kg

= **$+9.58 \times 10^{7}$ C kg^{-1}**

This is about 1/1800 the specific charge on an electron.

14D.01.9

It is possible.
Alpha particles can be accelerated using an electric field,
since they are positively charged.

14D.01.10

(a)

e/m = (2 × 1.60 × 10^{-19} C) ÷ (4 × 1.67 × 10^{-27} kg) = **4.79 × 10^{7} C kg^{-1}**

(b)

$$V = (e/m) \times 2B^2d^2$$

V = 4.79 × 10^{7} C kg^{-1} × 2 × $(0.145 \text{ T})^2$ × $(0.075 \text{ m})^2$ = 11331 V = **11300 V** (3 s.f.)

Tutorial 14D.02

14D.02.1

Gravity acts downwards.

Force of electrostatic attraction acts upwards.

The resultant force is zero.

Because the two forces are equal and opposite.

14D.02.2

We could measure the diameter of the sphere
And from that work out the volume.
Then use mass = density × volume to get the mass
Then multiply the mass by g to get the weight.

14D.02.3

Weight acting downwards
is balanced
By viscous drag acting upwards.
Resultant force is zero.

14D.02.4

$$F = mg = 6\pi\eta r v$$

F = 1 × 10^{-3} kg × 9.8 m s^{-2} = 0.0098 N

0.0098 N = 6 × π × 0.36 N s m^{-2} × 0.5 × 10^{-3} m × v

v = 0.0098 N ÷ 3.39 × 10^{-3} N s m^{-1} = 2.95 m s^{-1} = **3.0 m s^{-1}** (to 2 s.f.)

14D.02.5

(a)

Speed = 2.0 × 10^{-3} m ÷ 18 s = **1.11 × 10^{-4} m s^{-1}** = 1.1 × 10^{-4} m s^{-1} (2.s.f.)

(b)

$$r^2 = \frac{9hv}{2rg}$$

= (9 × 1.80 × 10^{-5} N s m^{-2} × 1.11 × 10^{-4} m s^{-1}) ÷ (2 × 970 kg m^{-3} × 9.8 m s^{-2})

= 9.46 × 10^{-13} m^2

r = **9.73 × 10^{-7} m** = 9.7 × 10^{-7} m (2 s.f.)

(c) Find out the mass

Mass = 4/3 × π × (9.73 × 10^{-7} m)^3 × 970 kg m^{-3} = 3.74 × 10^{-15} kg

Weight = 3.74 × 10^{-15} kg × 9.8 m s^{-2} = 3.67 × 10^{-14} N

$F = Eq$ = weight

57000 N C^{-1} × q = 3.67 × 10^{-14} N

q = **6.43 × 10^{-19} C** = 6.4 × 10^{-19} C (2 s.f.)

(d)

This is 4 times the electronic charge.

It tells us the charge is quantised (goes up in whole number multiples)

Tutorial 14D.03

14D.03.1

The velocity in the material is greater than the velocity in air.
This is not consistent
Since we know that the speed of light is less in a material than in air.

14D.03.2

The wavefront appears to be turned
upside down.

14D.03.3

It is consistent.
We observe that the speed of light in a material is less than
the speed of light in air.

14D.03.4

Newton's corpuscular theory was more accepted
because of his reputation.
There was not much evidence to support wave theory at the time.
Waves could not travel in a vacuum.
People thought that all waves needed a material to travel in.

14D.03.5

Waves that are of same wavelength;
Same frequency;
Same phase relationship.

14D.03.6

Interference fringes could be explained by waves.
Light travelled slower in glass than air as predicted by wave model.
This contradicted the particle model which said light would go faster.

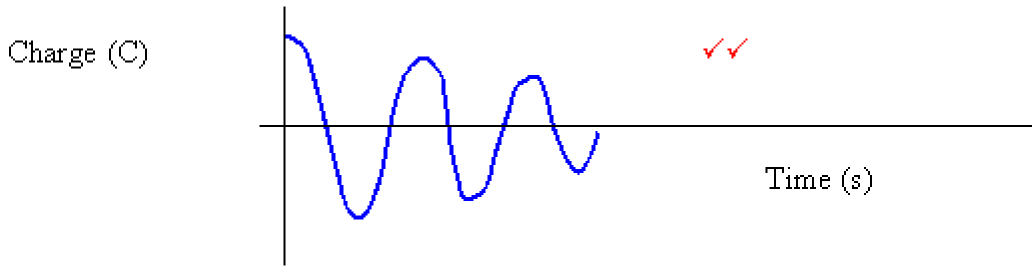
14D.03.7

$$c = \frac{1}{\sqrt{(4 \times \pi \times 10^{-7}\ \text{H m}^{-1} \times 8.85 \times 10^{-12}\ \text{C}^2\ \text{N}^{-1}\ \text{m}^{-2})}} = \frac{1}{\sqrt{(1.11 \times 10^{-17}\ \text{s}^2\ \text{m}^{-2})}}$$

$$= \frac{1}{3.33 \times 10^{-9}\ \text{s m}^{-1}}$$

c = **2.99×10^8 m s^{-1}**

14D.03.8



Tutorial 14D.04

14D.04.1

The plates would discharge
because the bright red light would have big waves
which carried lots of energy.

14D.04.2

E: photon energy (J);
h: Planck's constant (6.63×10^{-34} Js);
f: frequency of the radiation (Hz).

14D.04.3

More photoelectrons will be released.
No contradiction.
as waves of bigger amplitude have more energy;
to remove more photoelectrons.

14D.04.4

(a)

$$E = 4.9 \text{ eV} \times 1.6 \times 10^{-19} \text{ J eV}^{-1} = \underline{\mathbf{7.84 \times 10^{-19} \text{ J}}}$$

(b)

Work out photon energy, $E = hf = 6.63 \times 10^{-34} \text{ Js} \times 1.7 \times 10^{15} \text{ Hz}$

$$E = 11.2 \times 10^{-19} \text{ J}$$

$$E_k = hf - \Phi = 11.2 \times 10^{-19} \text{ J} - 7.84 \times 10^{-19} \text{ J} = \underline{\mathbf{3.36 \times 10^{-19} \text{ J}}}$$

(c)

$$Ek = eV_s \Rightarrow V_s = Ek/e$$

$$V_s = 3.36 \times 10^{-19} \text{ J} \div 1.6 \times 10^{-19} \text{ C}$$

$$= \underline{\mathbf{2.1 \text{ V}}}$$

(d)

$$v^2 = 2Ek/m = 2 \times 3.36 \times 10^{-19} \text{ J} \div 9.11 \times 10^{-31} \text{ kg} = 7.38 \times 10^{11} \text{ m}^2 \text{ s}^{-2}$$

$$v = \underline{\mathbf{8.6 \times 10^5 \text{ m s}^{-1}}}$$

14D.04.5

$$\lambda = \frac{h}{\sqrt{2meV}}$$

$$\lambda = \frac{6.63\times10^{-34}}{\sqrt{2\times9.11\times10^{-31}\times1.6\times10^{-19}\times40}} = \frac{6.63\times10^{-34}}{\sqrt{1.17\times10^{-47}}} = \frac{6.63\times10^{-34}}{3.42\times10^{-24}}$$

$$\lambda = 1.9\times10^{-10}\ \text{m}$$

14D.04.6

$$\lambda = \frac{h}{\sqrt{2meV}} = \frac{6.63\times10^{-34}}{\sqrt{2\times9.11\times10^{-31}\times1.6\times10^{-19}\times1\times10^{5}}}$$

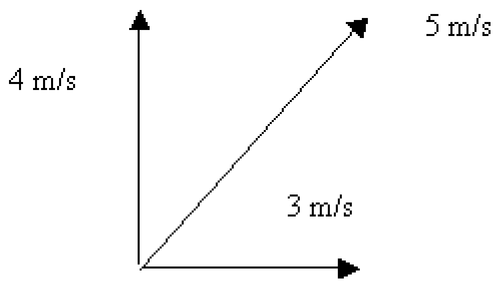
$$\lambda = 3.88\times10^{-12}\ \text{m}$$

14D.04.7

Electrons lose momentum going through the sample.
Reduced momentum means a longer wavelength, hence reduced resolution.
The lenses may not focus correctly
Due to variations in the current.

Tutorial 14D.05

14D.05.1



(a)

Velocity is 4 m s^{-1} (since it's a 3, 4, 5 triangle).

(b)

Time = distance ÷ speed = 400 m ÷ 4 m s^{-1} = 100 s

(c)

Velocity = 5 m s^{-1} – 3 m s^{-1} = 2 m s^{-1}

Time = 200 m ÷ 2 m s^{-1} = 100 s

(d)

Velocity = 5 m s^{-1} + 3 m s^{-1} = 8 m s^{-1}

Time = 200 m ÷ 8 m s^{-1} = 25 s

(e)

Boat X wins

By 25 s

14D.05.2

Unless a resultant force acts on a body,
its velocity will not change.
It will remain at rest or moving at a constant speed.

14D.05.3

Zero force has been applied to the trolley
So there should be zero change in motion.

14D.05.4

The path appears curved
Which would need a force to be applied.

14D.05.5

It's an accelerating frame of reference
Because it's circular motion
And the acceleration is centripetal

14D.05.6

Light travels very fast
So you need a very long path to get a reasonable time interval.

14D.05.7

No
The light waves are travelling in a material (glass)
So will be slower, as glass has a refractive index of 1.5.

So, its speed will be about 2.0×10^{8} m s^{-1}.

Tutorial 14D.06

14D.06.1

A: 300 m/s

B: 330 m/s

C: 360 m/s

14D.06.2

It is a fundamental particle

A lepton

Quite massive (similar mass to a meson)

Has a negative charge.

14D.06.3

(a) time = distance ÷ speed

$$= \frac{2000\ \text{m}}{0.996 \times 3.0 \times 10^{8}\ \text{m s}^{-1}} = \mathbf{6.69 \times 10^{-6}\ s}$$

(b) This represents

6.69×10^{-6} s ÷ 2.20×10^{-6} s = **3.04** half-lives.

(c) Intensity = $I_0/2^{3.04}$

= 1/8.22 = **0.12** of intensity at A (12 %)

(d) Half-life of moving muons:

$v^2/c^2 = 0.996^2 = 0.992016$

$1 - 0.992016 = 7.98 \times 10^{-3}$

$\sqrt{7.98 \times 10^{-3}} = 0.0893$

$\gamma = (0.0893)^{-1} = 11.2$

$t = 2.20 \times 10^{-6} \times 11.2 = \mathbf{2.46 \times 10^{-5}\ s}$

(e) Number of moving half-lives = $6.69 \times 10^{-6} \div 2.46 \times 10^{-5}$ = **0.272**

(f) Intensity at B = I at A ÷ $2^{0.272}$

= I at A ÷ 1.207 = **0.83** (= 83 %)

14D.06.4

$$v^2/c^2 = 0.98^2 = 0.9604$$

$$1 - 0.9604 = 0.0396$$

$$\alpha = \sqrt{0.396} = 0.198$$

$$\text{Length} = 60 \times 0.198 = \underline{\mathbf{11.93\ m}}$$

14D.06.5

$$l/l_0 = 0.8 = \sqrt{(1 - v^2/ (3.0 \times 10^8)^2)}$$

$$0.64 = 1 - v^2/ (3.0 \times 10^8)^2$$

$$-0.36 = - v^2/ (3.0 \times 10^8)^2$$

$$v^2 = 0.36 \times 9.0 \times 10^{16}\ \text{m}^2\ \text{s}^{-2} = 3.24 \times 10^{16}\ \text{m}^2\ \text{s}^{-2}$$

$$v = \underline{\mathbf{1.8 \times 10^8\ m\ s^{-1}}}\ (= 0.6\ c)$$

14D.06.6

(a)

$v^2/c^2 = 0.996^2 = 0.992016$

$1 - 0.9992016 = 7.984 \times 10^{-3}$

$\sqrt{7.984 \times 10^{-3}} = 0.08935$

Height of mountain = 2000 m × 0.08935 = **178.7 m**

(b)

Time taken = 178.7 ÷ 0.996 × 3.0 × 10^8 = **5.98×10^{-7} s**

(c)

Number of half-lives = 5.98×10^{-7} s ÷ 2.2×10^{-6} s

= **0.272**

(d)

Intensity at B = I at A ÷ $2^{0.272}$

= *I* at A ÷ 1.207 = 0.83 (= 0.83 %)

14D.06.7

$v^2/c^2 = 0.998^2 = 0.996004$

$1 - 0.996004 = 0.003996$

$\sqrt{0.003996} = 0.063214$

$\gamma = (0.063214)^{-1} = 15.82$

Mass = 9.11×10^{-31} kg × 15.82 = **1.44×10^{-29} kg**

14D.06.8

Circumference of semi-circle = πr = $\pi \times 4 \times 10^8$ m = 12.57×10^8 m

This is done in 1 s, so speed of sweep = **12.57×10^8 m s^{-1}**

14D.06.9

Use:

$$eV = \frac{1}{2}mv^2$$

Rearranging:

$$V = \frac{mv^2}{2e}$$

V = (9.11×10^{-31} kg × (3.00×10^8 m s^{-1})^2) ÷ ($2 \times 1.60 \times 10^{-19}$ C)

V = **2.56×10^5 V**

14D.06.10

Rest energy = 1.67×10^{-27} kg × 9.0×10^{16} m^2 s^{-2} = 1.50×10^{-10} J

Total energy of the proton is given by:

$$E = \frac{m_0 c^2}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$

$v^2/c^2 = 0.95^2 = 0.9025$

$1 - 0.9025 = 0.0975$

$\sqrt{0.0975} = 0.312$

Total energy = 1.50×10^{-10} J ÷ 0.312 = 4.80×10^{-10} J

Kinetic energy = 4.80×10^{-10} J - 1.50×10^{-10} J = 3.30×10^{-10} J

Voltage = 3.30×10^{-10}J ÷ 1.6×10^{-19} C = **2.06×10^9 V**

14D.06.11

(a)

1 C = 6.25×10^{18} electrons.

Q = (1.2×10^{15} electrons) ÷ 6.25×10^{18} electrons C^{-1} = **1.92×10^{-4} C**

(b)

Energy = $1.2 \times 10^{15} \times 2.50 \times 10^6$ V × 1.60×10^{-19} C = 480 J

Temperature change = 480 J ÷ (0.100 kg × 900 J kg^{-1} K^{-1}) = **5.3 K**

14D.06.12

Use:

$$u = \frac{v + u'}{1 + \frac{vu'}{c^2}}$$

$vu'/c^2 = (0.40\ c \times 0.125\ c) \div c^2 = 0.05$

Downstairs value = 1 + 0.05 = 1.05

$u = (0.40\ c + 0.125\ c) \div 1.05 = 0.50\ c$ (which is what we had before, so the stationary observer's measurement was correct.)